# Aerospace Combustion

## Lecture 10:

## Combustion Theory: Non-Premixed Flames

# Content

- Non-Premixed Flames
  - Laminar-Diffusion Flames
    - Jet Flames
    - Counter-flow Flames
    - Burke-Schuman Solution
  - Turbulent Diffusion Flames
    - Jet Flames

# Non-Premixed Flames

## Laminar Diffusion Flames

### Basic Characteristics

- Propellants are supplied separately into the combustor
- In case moderate mixing in the shear layer between propellants, reaction takes place in the thin reaction layer
- Propellants are mainly transported by diffusion processes (concentration, temperature gradients)
- Few real applications (Bunsen burner, candle) studied in labs to measure specific flame features, i.e. laminar flame velocity

laminar

turbulent

# Laminar Diffusion Flames

## Basic Assumptions

- Fast chemistry → no details of chemical kinetics required
- If characteristic time scales of flow and chemistry are similar
  → problems with ignition and extinction and in formation of by-products (pollutants)

→ Formulation required which includes main kinetic features ( reduced mechanisms )

→ assume equal diffusivities of chemical species:

$$Le_i = \frac{\lambda_i}{(c_{p,i}\rho D_i)} = 1;\ I = 1,2 \ldots k$$

$$Le = \frac{\alpha}{D} = \frac{\lambda}{c_p \rho D}$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### 1-D Planar Flame

$$Pe = \frac{uL}{\alpha}; Le_O = \frac{\alpha}{D_O}; Le_F = \frac{\alpha}{D_F};$$

$$\tilde{x} = \frac{x}{L}; S = \frac{v_O M_O}{v_F M_F} \frac{Y_{F,0}}{Y_{O,0}}$$

$$\Rightarrow \frac{Y_O}{Y_{O,0}} = \frac{1 - e^{PeLe_O(\tilde{x} - \tilde{x}_f)}}{1 - e^{PeLe_O(1 - \tilde{x}_f)}}; \quad \frac{Y_F}{Y_{F,0}} = \frac{1 - e^{PeLe_F(\tilde{x} - \tilde{x}_f)}}{1 - e^{-PeLe_F \tilde{x}_f}};$$

Flame Position $x_f$ ?: Let's assume a 1-step reaction: $v_F \text{ fuel} + v_O \text{ oxidizer} \rightarrow \text{products}$

at $x = x_f$, the ratio of oxidizer and fuel has to be stoichiometric;

$$\rho D_F \left.\frac{dY_F}{dx}\right|_{x=x_{f+}} = \frac{\upsilon_F M_F}{\upsilon_O M_O} \rho D_O \left.\frac{dY_O}{dx}\right|_{x=x_{f-}}$$

$$\frac{D_F Y_{F,0} e^{-(u/D_F)x_f}}{1 - e^{-(u/D_F)x_f}} \frac{u}{D_F} e^{(u/D_F)x_F} = -\frac{\upsilon_F M_F}{\upsilon_O M_O} \frac{D_O Y_{O,0} e^{-(u/D_O)x_f}}{1 - e^{(u/D_O)(L - x_f)}} \frac{u}{D_O} e^{(u/D_O)x_F} \Rightarrow$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### 1-D Planar Flame: Flame Position

$$\Rightarrow \frac{Y_{F,0}}{1-e^{-(u/D_F)x_f}} = -\frac{\upsilon_F M_F}{\upsilon_O M_O}\frac{Y_{O,0}}{1-e^{(u/D_O)(L-x_f)}} \quad \Rightarrow \quad S\left(1-e^{PeLe_O(1-\tilde{x}_f)}\right) = -\left(1-e^{-PeLe_F\tilde{x}_f}\right)$$

$$\Rightarrow \quad Se^{PeLe_O(1-\tilde{x}_f)} + e^{-PeLe_F\tilde{x}_f} = S+1$$

no analytical solution possible, however

Solutions for Special Cases: mass transport << energy transport: Pe → *0* since exp(Pe) ~ 1 + Pe

$$\Rightarrow \quad S\left(1+PeLe_O(1-\tilde{x}_f)\right) + \left(1-PeLe_F\tilde{x}_f\right) \approx S+1$$

$$\tilde{x}_f \approx \left(1+\frac{Le_F}{SLe_O}\right) \Rightarrow x_f = L\left(1+\frac{\upsilon_F M_F}{\upsilon_O M_O}\frac{Y_{O,0}}{Y_{F,0}}\frac{Le_F}{Le_O}\right)^{-1}$$

# Laminar Diffusion Flames

## 1-D Planar Flame: Flame Position

Solutions for Special Cases: $Le_O = Le_F = 1$, diffusive transport of energy and mass identical for both fluids

$$Se^{PeLe_O(1-\tilde{x}_f)} + e^{-PeLe_F\tilde{x}_f} = S+1; \quad \Rightarrow \quad \left(Se^{Pe}+1\right)e^{-Pe\tilde{x}_f} = S+1 \quad \Rightarrow e^{-Pe\tilde{x}_f} = \frac{1+S}{1+Se^{Pe}}$$

$$\Rightarrow \quad -Pe\tilde{x}_f = \ln\left(\frac{1+S}{1+Se^{Pe}}\right); \quad \Rightarrow x_F = -\frac{L}{Pe}\ln\left(\frac{1+S}{1+Se^{Pe}}\right)$$

Solutions for Special Cases: Pe→∞, mass transport >> energy transport

$$Se^{PeLe_O(1-\tilde{x}_f)} + e^{-PeLe_F\tilde{x}_f} = S+1; \quad \Rightarrow$$

$$\left(1-\tilde{x}_f\right) = \frac{1}{PeLe_O}\ln\left(\frac{1+S}{S}\right); \quad \Rightarrow x_f = L\left(1-\frac{1}{PeLe_O}\ln\left(\frac{1+S}{S}\right)\right)$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### 1-D Planar Flame: Temperature Profiles

Fuel Side:

$$u\frac{dT}{dx}-\frac{\lambda}{\rho c_p}\frac{d^2T}{dx^2}=0;\ T=T_{F,0}\Big|_{x=0};T=T_f\Big|_{x=x_f}$$

$$T(x)=T_{F,0}+\left(T_f-T_{F,0}\right)\frac{1-e^{(u/\alpha)x}}{1-e^{(u/\alpha)x_f}}=$$

$$=T_{F,0}+\left(T_f-T_{F,0}\right)\frac{1-e^{Pe\tilde{x}}}{1-e^{Pe\tilde{x}_f}}$$

Oxidizer Side:

$$u\frac{dT}{dx}-\frac{\lambda}{\rho c_p}\frac{d^2T}{dx^2}=0;\ T=T_{O,0}\Big|_{x=L};T=T_f\Big|_{x=x_f}$$

$$T(x)=T_f-\left(T_f-T_{O,0}\right)\frac{1-e^{(u/\alpha)\left(x-x_f\right)}}{1-e^{(u/\alpha)(L-x_f)}}=$$

$$=T_f-\left(T_f-T_{O,0}\right)\frac{1-e^{Pe\left(\tilde{x}-\tilde{x}_f\right)\tilde{x}}}{1-e^{Pe\left(1-\tilde{x}_f\right)}}$$

At flame front fuel and oxidizer side temperatures have to be identical

→ <u>Conductive heat flux away flame front has to equal on both sides and equal as well to the local heat release</u>

# Laminar Diffusion Flames

## 1-D Planar Flame: Temperature Profiles

<u>Special Case Pe→0</u>:

1. Temperature independent on Pe number as expected
2. Decreasing Le number similar effect as increasing reactant concentration

→ This is entirely different than for premixed flames where adiabatic flame temperature is independent on Le number

<u>Special Case $Le_F = Le_O = 1$</u>:

1. Temperature independent on Pe number

→ Convection doesn't influence temperature at flame front position

<u>Special Case Pe→∞</u>:

1. with increasing convective transport, both influence of oxidizer and fuel Le numbers decrease

## Laminar Diffusion Flames

### Spherical Jet Flame

### Assumptions:

Steady, axisymmetric, zero mean axial pressure gradient, convective transport only in x-direction, diffusive transport only in r-direction

Continuity

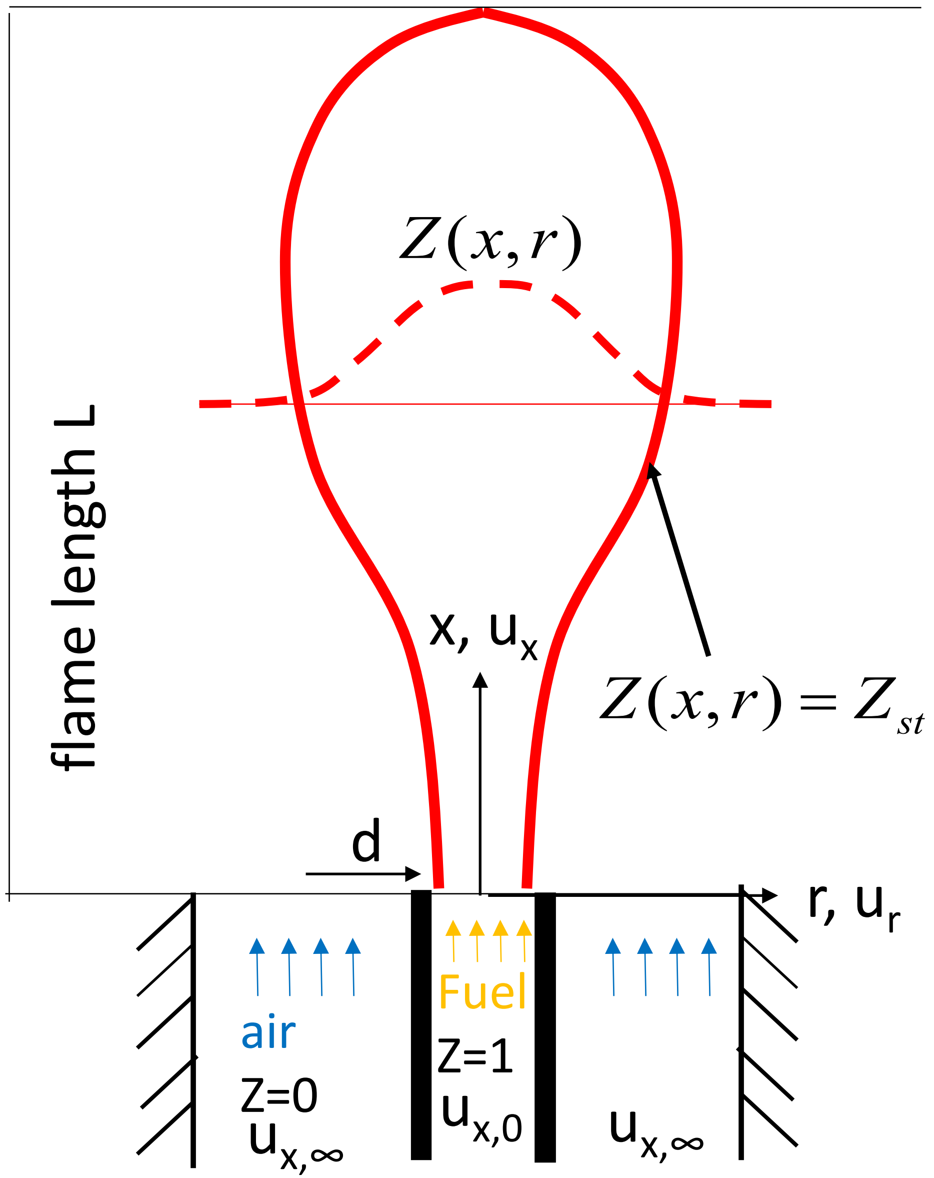
$$\frac{\partial(\rho u_x r)}{\partial x} + \frac{\partial(\rho u_r r)}{\partial r} = 0$$

Momentum in x-direction

$$\rho u_x r \frac{\partial u_x}{\partial x} + \rho u_r r \frac{\partial u_x}{\partial r} = -r \frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_x}{\partial r}\right)$$

Mixture fraction

$$\rho u_x r \frac{\partial Z}{\partial x} + \rho u_r r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\mu}{Sc} r \frac{\partial Z}{\partial r}\right)$$




Details of the derivation in the Annex

# Non-Premixed Flames

## Laminar Diffusion Flames

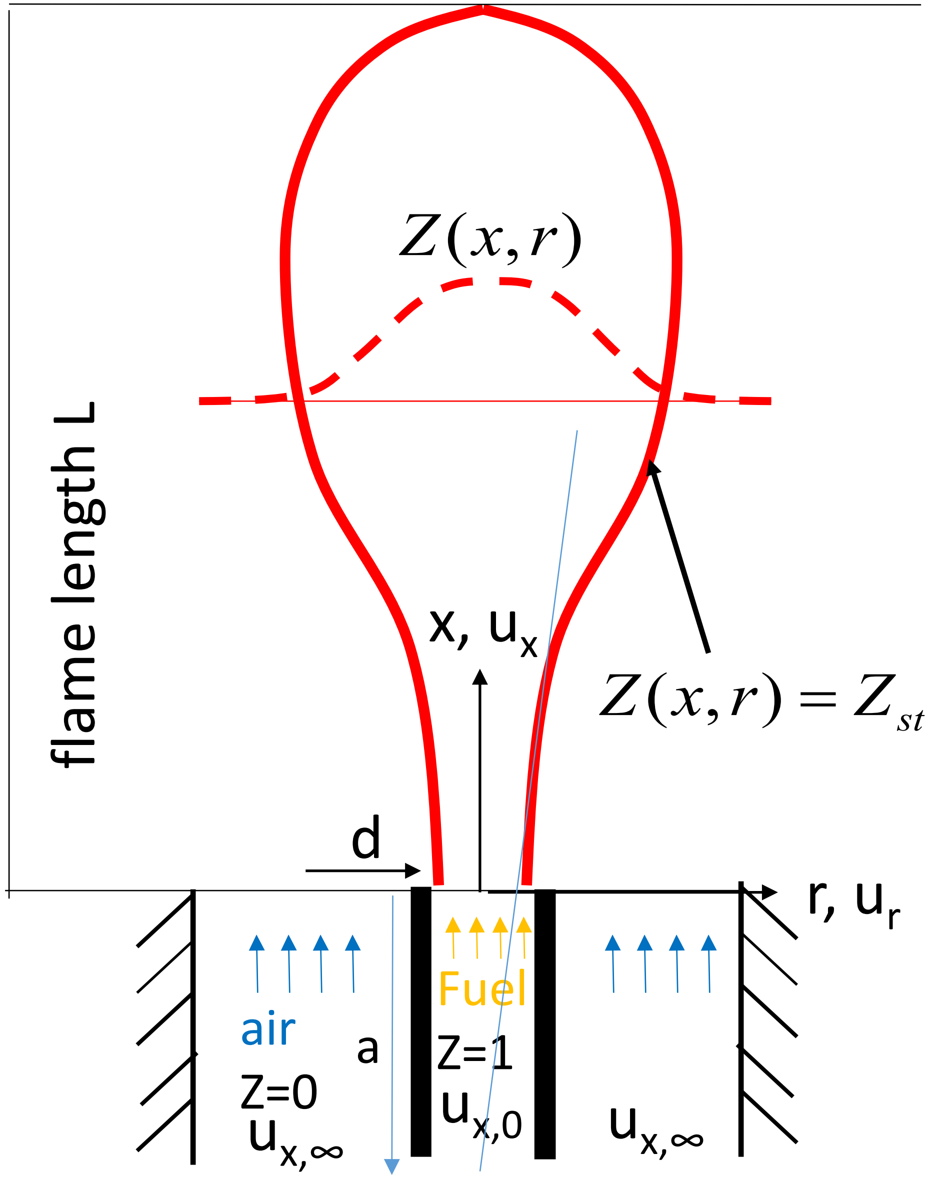
### Spherical Jet Flame

Flame contour form the condition $Z(x,r) = Z_a\omega(\eta) = Z_{st}$

Flame length at $Z(x,r=0)$ if $Z_a = Z_{st}$

$$Z_a(x) = \frac{1+2Sc}{32}\frac{\rho}{\rho_\infty}\frac{\mathrm{Re}}{C}\frac{d}{\phi}$$

$$L = \frac{1+2Sc}{32 Z_{st}}\frac{\rho}{\rho_\infty}\frac{u_0}{C}\frac{d^2}{\nu} - a$$



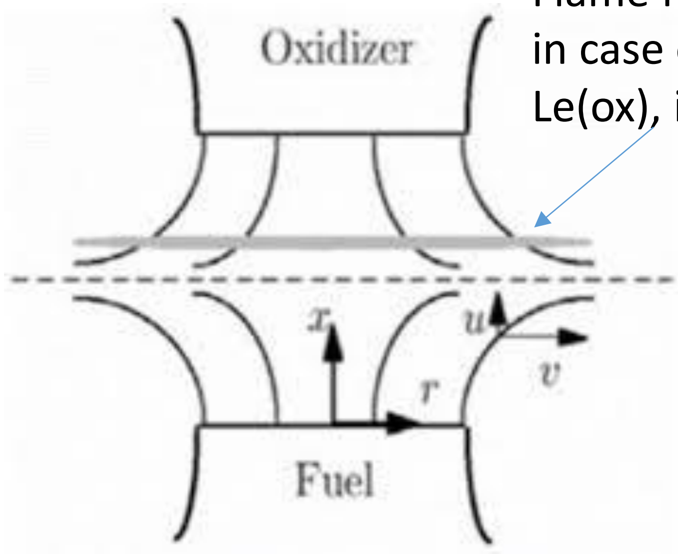
# Non-Premixed Flames

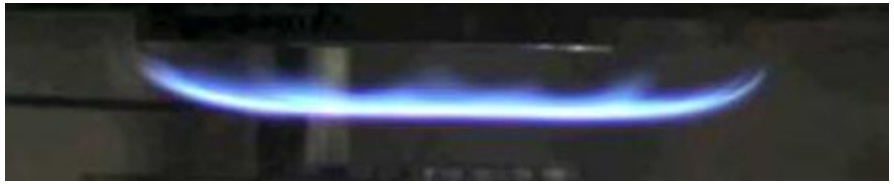
## Laminar Diffusion Flames

### Counter-Flow Flames

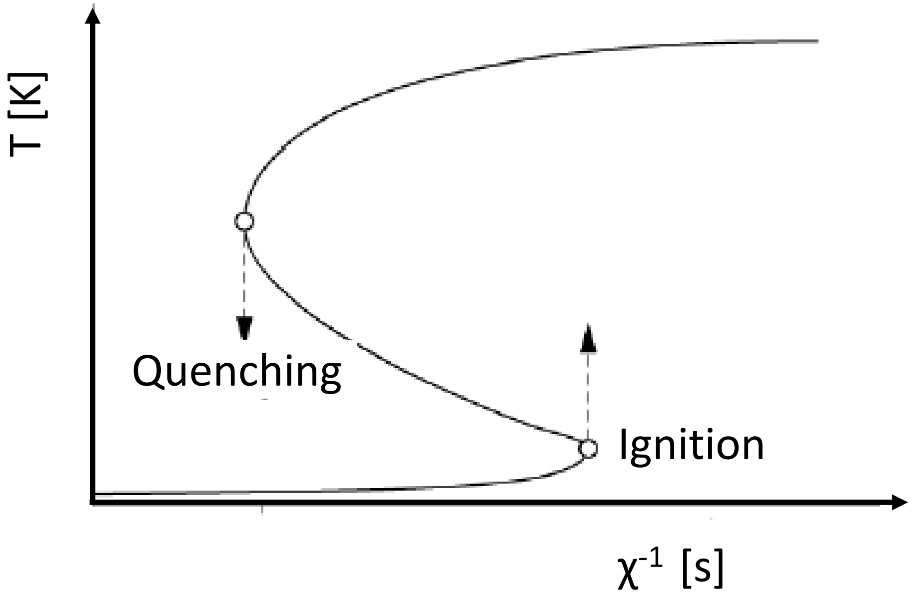
Experimental Setup to study the sensitivity of a flame to the application of shear forces.

Flame front position in case of Le(fuel) ǂ Le(ox), i.e. $H_2$ and $O_2$



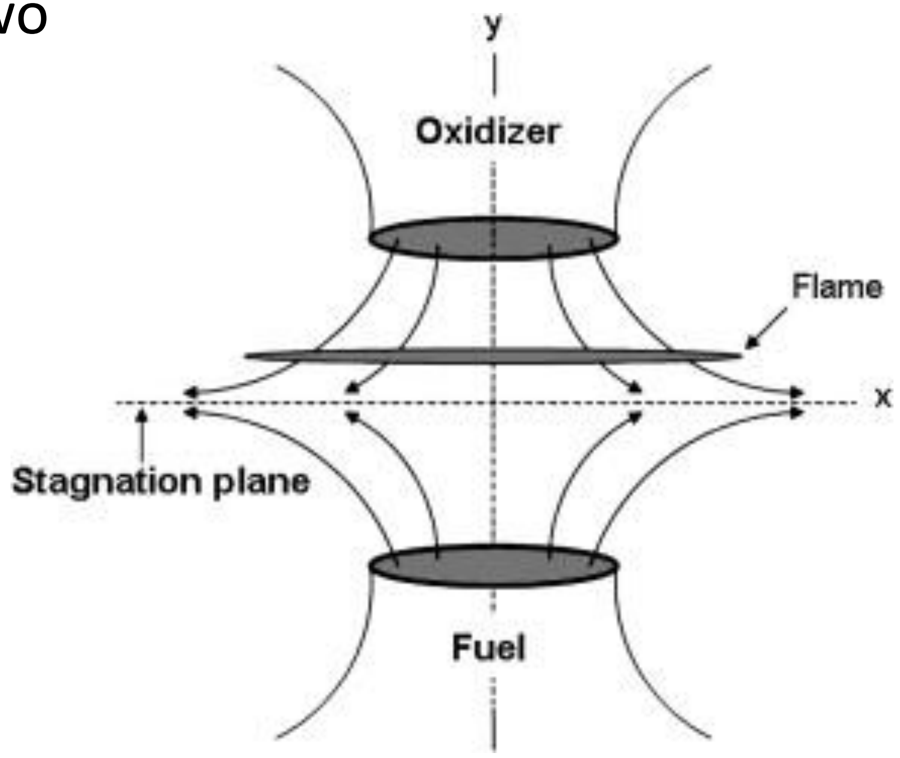
Of special interest are phenomena like quenching or ignition.
Solutions are only stable for the lower part of the curve prior to ignition and the upper part prior to extinction.



# Laminar Diffusion Flames

## Counter-Flow Flames

Assumptions: flow velocities large enough that the stagnation plane far away from the nozzles, the flame establishes between two potential flows and quite often on the ox-rich side.




Velocity gradient in the oxidizer $$a = \frac{\partial u_{y,\infty}}{\partial y};$$

yields velocities and mixture fraction

$$y \rightarrow \infty: \quad u_{y,\infty} = -ay; \qquad u_{x,\infty} = ax; \qquad Z = 0$$

Equal stagnation pressures of both streams yields

$$y \rightarrow -\infty: \; u_{y,-\infty} = -\left(\frac{\rho_\infty}{\rho_{-\infty}}\right) ay; \quad u_{x,-\infty} = \left(\frac{\rho_\infty}{\rho_{-\infty}}\right) ax; \quad Z = 1$$

# Laminar Diffusion Flames

## Counter-Flow Flames

Continuity, momentum and mixture fraction equations

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} = 0$$

$$\rho u_x \frac{\partial u_x}{\partial x} + \rho u_y \frac{\partial u_x}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u_x}{\partial y}\right)$$

$$\rho u_x \frac{\partial Z}{\partial x} + \rho u_y \frac{\partial Z}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial Z}{\partial y}\right)$$

Introduction of similarity transformations

$$\eta = \left(\frac{a}{\rho_\infty \mu_\infty}\right)^{1/2} \int_0^y \rho dy\,;$$

yields $F = \int_0^\eta F' d\eta;$

$$\frac{\partial}{\partial \eta}\left(C \frac{\partial F'}{\partial \eta}\right) + F \frac{\partial F'}{\partial \eta} + \frac{\rho_\infty}{\rho} - F'^2 = 0;$$

$$\frac{\partial}{\partial \eta}\left(\frac{C}{Sc} \frac{\partial Z}{\partial \eta}\right) + \mathrm{F} \frac{\partial Z}{\partial \eta} = 0$$

with *C* and *Sc* the Chapman-Rubesin and Schmidt number, respecivelly

$$C = \frac{\rho \mu}{\rho_\infty \mu_\infty}; \quad Sc = \frac{\mu}{\rho D}$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### Counter-Flow Flames

in terms of non-dimensional stream function $\mathrm{F} = \frac{\rho u_y}{\sqrt{a \rho_\infty \mu_\infty}}$ normalized tangential velocity $F' = \frac{u_x}{ax} 1;$

with boundary conditions

$$\eta = +\infty : \quad F' = 1; Z = 0$$

$$\eta = -\infty : \quad F' = \left(\rho_\infty / \rho_{-\infty}\right)^{1/2}; Z = 1$$

$$\rightarrow \quad Z = \frac{1}{2} \frac{I(\infty) - I(\eta)}{I(\infty)};$$

with *I(η)* defined as

$$I(\eta) = \int_0^\eta \frac{Sc}{C} e^{\left[ -\int_0^\eta F \frac{Sc}{C} d\eta \right]} d\eta$$

# Laminar Diffusion Flames

## Counter-Flow Flames

For $C = 1$, $\rho = const.$, $F=\eta$ satisfies

$$\frac{\partial}{\partial \eta}\left( C \frac{\partial F'}{\partial \eta} \right) + F \frac{\partial F'}{\partial \eta} + \frac{\rho_\infty}{\rho} - F'^2 = 0$$

and

$$Z = \frac{1}{2} erfc\left(\eta / \sqrt{2}\right)$$

the instantaneous scalar dissipation rate is

$$\chi = 2D\left(\frac{\partial Z}{\partial y}\right)^2 = 2\left(\frac{C}{Sc}\right) a \left(\frac{\partial Z}{\partial \eta}\right)^2$$

With $\eta = \left(\frac{a}{\rho_\infty \mu_\infty}\right)^{1/2} \int_0^y \rho dy$; and $C = \frac{\rho\mu}{\rho_\infty \mu_\infty}$; $Sc = \frac{\mu}{\rho D}$

# Laminar Diffusion Flames

## Counter-Flow Flames

Evaluation of scalar dissipation rate which led to $Z = \frac{1}{2} erfc\left(\eta / \sqrt{2}\right)$

yields $$\chi = \frac{a}{\pi} e\left(-\eta^2(Z)\right) = \frac{a}{\pi} e\left(-2[erfc^{-1}(2Z)]^2\right)$$

For small Z $$\frac{dZ}{d\eta} = -\frac{1}{2}\frac{dI}{d\eta}\frac{1}{I(\infty)} = \frac{dI}{d\eta}\frac{Z}{I(\infty)-I(\eta)} = -\frac{Sc}{C} FZ$$

hence, in terms of velocity gradient , the scale dissipation rate becomes $\chi = 2aFZ^2\left(\frac{Sc}{C}\right)$

which means that for small Z, the dissipation rate increases with $Z^2$

Neglecting the unsteady term in $\rho\frac{\partial T}{\partial t} - \rho\frac{\chi_{st}\partial^2 T}{2\partial Z^2} = \sum_{l=1}^{r}\frac{Q_l}{c_p} + \frac{\dot{q}R}{c_p} + \frac{1}{c_p}\frac{\partial p}{\partial_t}$ yields

# Non-Premixed Flames

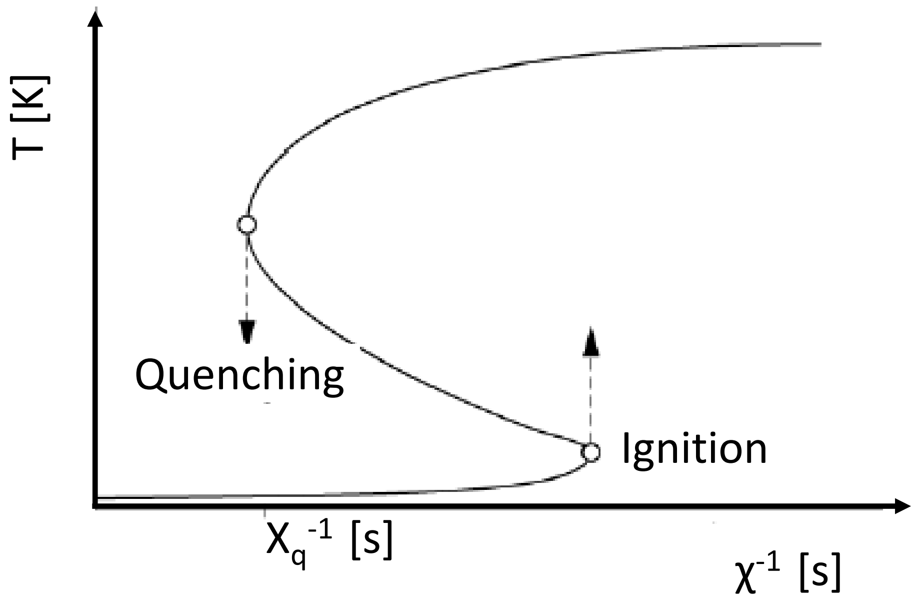
## Laminar Diffusion Flames

### Counter-Flow Flames

Steady State Combustion/Quenching with One Step Chemistry, large activation energy, constant pressure and negligible radiative losses

$$-\rho \frac{\chi_{st}\partial^2 T}{2\partial Z^2} = \frac{Q}{c_p}\omega \quad \text{with} \quad \omega = B\frac{\rho Y_F}{W_F}\frac{\rho Y_{O_2}}{W_{O_2}} e^{\frac{-E_a}{RT}}$$

Analyzing the upper branch of the strain rate curve for large Damköhler numbers and activation energies which corresponds to a complete combustion in an infinitely thin flame sheet around $Z = Z_{st}$.



## Laminar Diffusion Flames

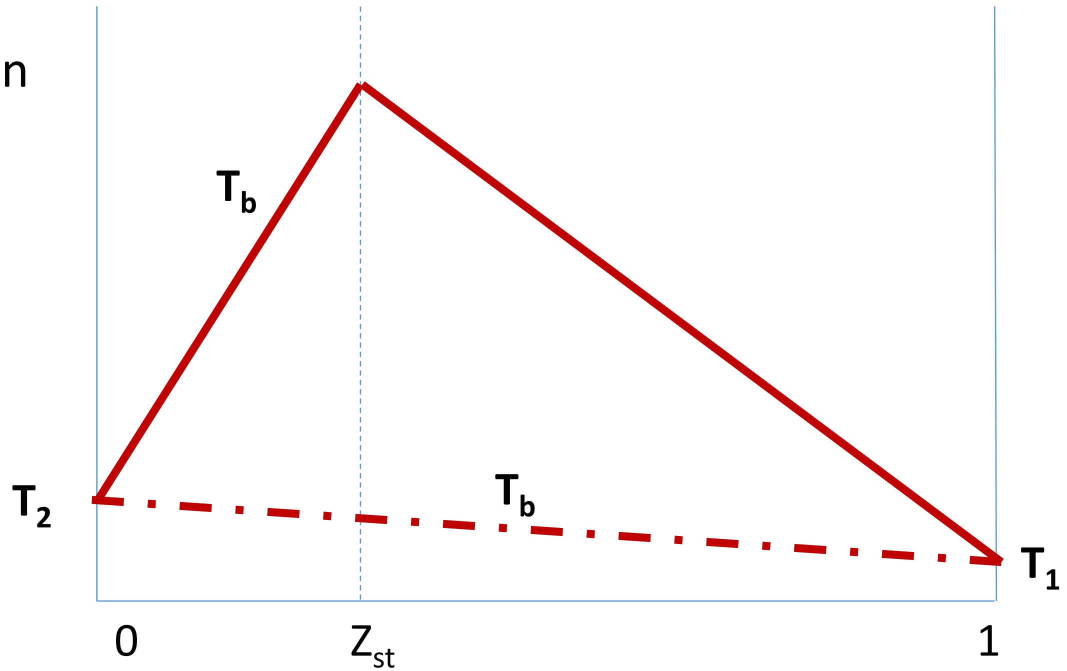
### Counter-Flow Flames: Burke-Schumann Solution (1-step chemistry)

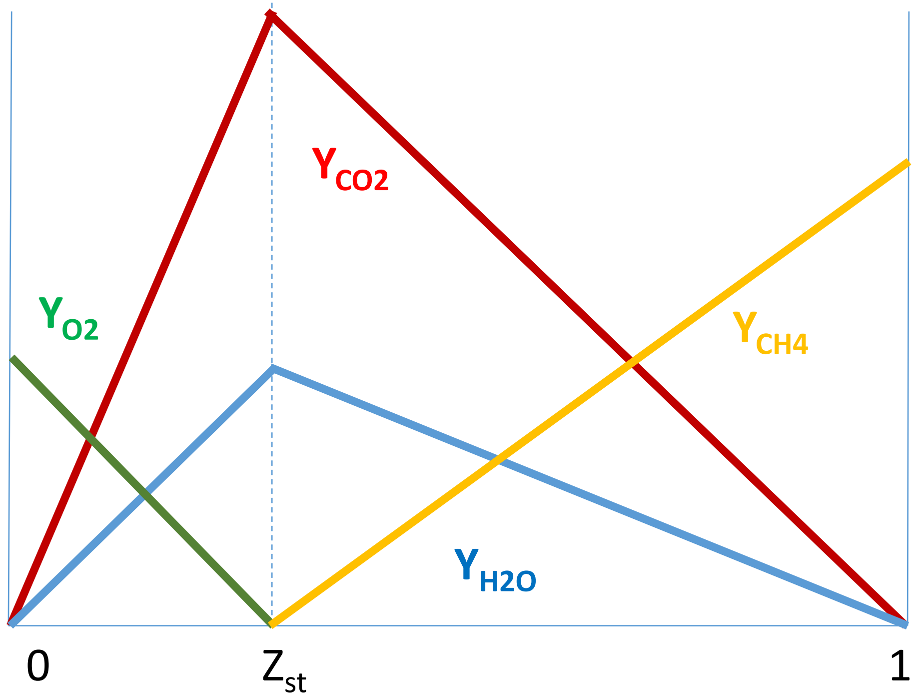
lean side $$T(Z) = T_u(Z) + \frac{QY_{CH_4,1}Z}{c_p \nu'_{CH_4} W_{CH_4,1}}; Y_{CH_4} = 0, Y_{O_2} = Y_{O_2,2}\left(1 - \frac{Z}{Z_{st}}\right)$$

rich side $$T(Z) = T_u(Z) + \frac{QY_{O_2,2}}{c_p \nu'_{O_2} W_{O_2}}(1-Z); Y_{O_2} = =; Y_{CH_4} = Y_{CH_4,1}\left(\frac{Z - Z_{st}}{1 - Z_{st}}\right)$$

Large activation energy, constant pressure and negligible radiation losses

For $c_p = const.$, temperature, and fuel, oxidizer, and product mass fractions rare piecewise linear functions of $Z$



# Laminar Diffusion Flames

## Counter-Flow Flames

With the reaction rate

$$\omega = B \frac{\rho Y_F}{W_F} \frac{\rho Y_{O_2}}{W_{O_2}} e^{\frac{-E_a}{RT}}$$

and

$$\frac{Q}{c_p} = \frac{(T_{st} - T_u)\nu'_F W_F}{Y_{F,1} Z_{st}}$$

we can show that

$$-\rho \frac{\chi_{st} \partial^2 T}{2 \partial Z^2} = \frac{Q}{c_p} \omega$$

is able to describe quenching of a diffusion flame.

Let $T_1 = T_2 = T_u$ we obtain

$$\frac{d^2 T}{dZ^2} = -\frac{2 B \nu'_F (T_{st} - T_u)}{\chi Y_{F,1} Z_{st} W_{O_2}} Y_F Y_{O_2} e^{\frac{-E_a}{RT}}$$

# Laminar Diffusion Flames

## Counter-Flow Flames

Expand temperature, fuel and oxygen mass fraction around $Z_{st}$ to with ε as small parameter

$$T = T_{st} - \varepsilon\left(T_{st} - T_u\right)y$$

$$Y_F = Y_{F,1}\varepsilon(Z_{st}y + \xi)$$

$$Y_{O_2} = Y_{O_2,2}\varepsilon((1 - Z_{st})y - \xi)$$

The exponential term of the reaction rate can be expanded to

$$e^{\frac{-E_a}{RT}} = e^{\frac{-E_a}{RT_{st}}} = e^{-Ze\varepsilon y}$$

where Ze is the Zeldovich number: $Ze = \dfrac{E_a\left(T_{st} - T_u\right)}{RT_{st}^2}$

## Laminar Diffusion Flames

### Counter-Flow Flames

Expansion of all quantities in $\frac{d^2T}{dZ^2} = -\frac{2B\nu'_F(T_{st}-T_u)}{\chi Y_{F,1} Z_{st} W_{O_2}} Y_F Y_{O_2} e^{\frac{-Ea}{RT}}$ around their value at $T_{st}$

yields $$\frac{d^2y}{d\xi^2} = 2Da\varepsilon^3 \left(Z_{st}+\xi\right)\left((1-Z_{st})y-\xi\right)e^{(-Ze\varepsilon y)}$$ with $$Da = \frac{B\rho_{st}\nu'_{O_2}Y_{F,1}}{\chi_{st}W_F\left(1-Z_{st}\right)}e^{\frac{-Ea}{RT}}$$

the Damköhler number

This equation can be transformed into one similar to Linan's diffusion flame regime using

$$z = 2y\left(1-Z_{st}\right)Z_{st}$$

$$\gamma = 2Z_{st} - 1$$

$$\beta = \frac{Ze}{2Z_{st}(1-Z_{st})}$$

to finally yield $$\frac{d^2z}{d\xi^2} = Da\varepsilon^3\left(z^2-\xi^2\right)e^{(-\beta\varepsilon(z+\gamma\xi))}$$

# Laminar Diffusion Flames

## Counter-Flow Flames

Setting either $Da\varepsilon^3 = 1$ large Damköhler number expansion

or $\beta\varepsilon = 1$ large activation energy expansion

with $\delta = Da / \beta^3 \cong O(1)$

to finally get Linan's equation for the diffusion flame regime

$$\frac{d^2 z}{d\xi^2} = \left(z^2 - \xi^2\right) e^{\left(-\delta^{-1/3}\left(z + \gamma\xi\right)\right)}$$

Boundary conditions:

$$\frac{dz}{d\xi} = 1 \qquad \xi \to \infty$$

$$\frac{dz}{d\xi} = -1 \qquad \xi \to -\infty$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### Counter-Flow Flames

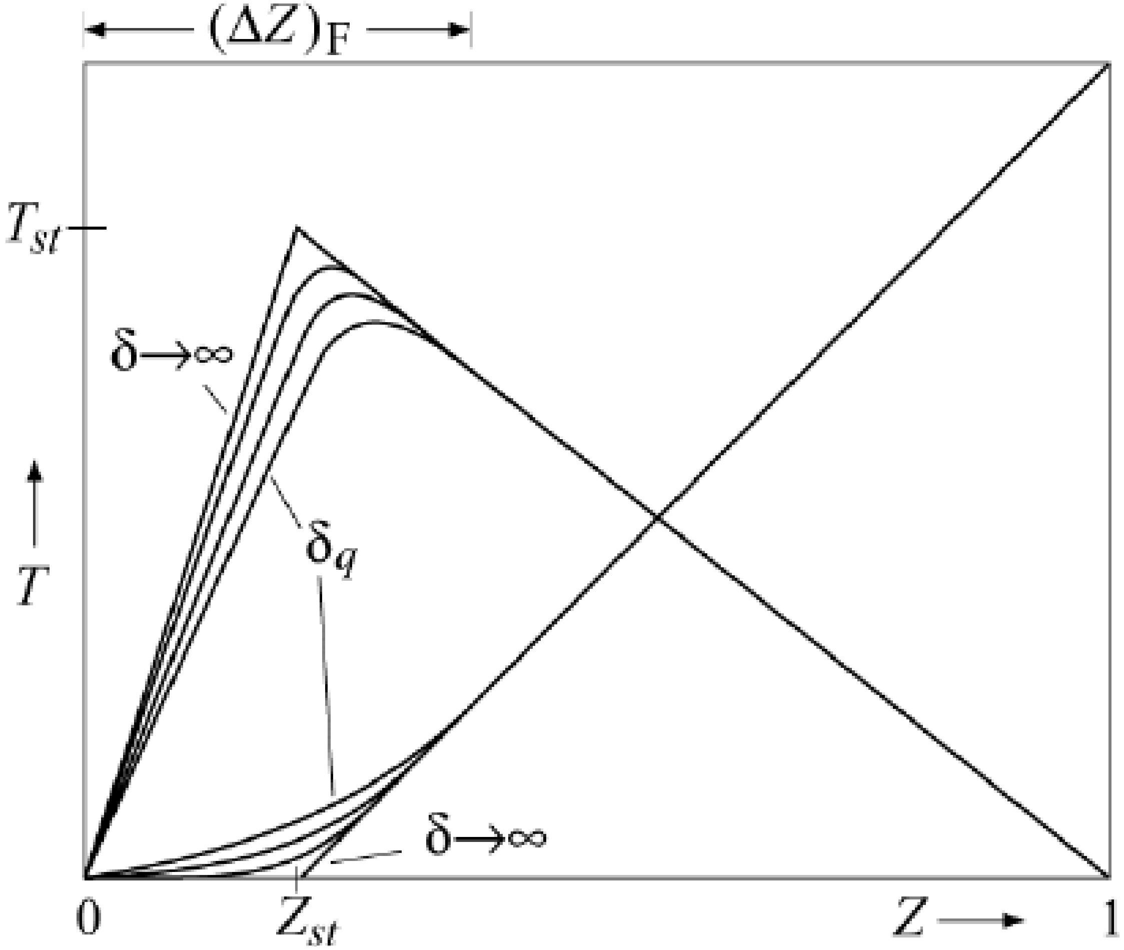
With $\delta$ being finite, a critical value $\delta_q$ can be determined for which quenching occurs.

Linan defines for pre-mixed flame regime to:

$$\delta_q = e(1-|\gamma|)$$

With the definition of Damköhler, a maximum dissipation rate $\chi_q$ can be determined:

$$\chi_q = \frac{8B\rho_{st}\nu'_{O_2}Y_{F,1}Z_{st}^3(1-Z_{st})^2}{\delta_q W_F Ze^3} e^{\frac{-E_a}{RT}}$$




Let's think of $\chi_{st}$ as an inverse diffusion time. For large $\chi_{st}$ heat conduction from the flame isn't outbalanced by local heat production and as a result local temperature decreases until finally at $\chi_{st} = \chi_q$.

# Non-Premixed Flames

## Laminar Diffusion Flames

### Time and Length Scales

Chemical time scale at extinction

$$t_c = \frac{Z_{st}^2 \left(1 - Z_{st}\right)^2}{\chi_q}$$

Chemical time scale at extinction for stoichiometric pre-mixed flame

$$t_c = \frac{\delta_q \left(\frac{\rho \lambda / c_p}{\rho}\right)_{st}}{2\left(\rho_u S_L\right)_{st}^2}$$

with

$$\rho_u S_L = \sqrt{\frac{2B\rho_b^2 R^2 T_b^4 \left(1 - Z_{st}\right)^2}{c_p \left(T_b - T_u\right)^2 E_a^2} e^{\frac{-E_a}{RT_b}} K}$$

$$K = \frac{\nu'_F Y_{O_2,b}}{M_{O_2}} + \frac{\nu'_{O_2} Y_{F,b}}{M_F} + \frac{2\nu'_{O_2} \nu'_F c_p R T_b^2}{\left(-\Delta H\right) E}$$

$$K = \begin{cases} \dfrac{\nu'_F Y_{O_2,b}}{M_{O_2}} & for \quad \phi << 1 \\ \dfrac{2\nu'_{O_2} \nu'_F c_p R T_b^2}{\left(-\Delta H\right) E_a} & for \quad \phi = 1 \\ \dfrac{\nu'_{O_2} Y_{F,b}}{M_F} & for \quad \phi >> 1 \end{cases}$$

# Non-Premixed Flames

## Laminar Diffusion Flames

### Counter-Flow Flames

This is a clear hint of a fundamental relation between a premixed flame and a diffusion flame at extinction.

1. For a diffusion flame, the heat conducted from the reaction zone towards the lean and the rich side just balances the heat generated by the reactions.
2. In a premixed flame, heat conducted towards the unburnt mixture is such that it balances the heat generation by the reaction for a particular burning velocity.

Diffusion flames can exist at lower scalar dissipation rates and therefore at lower characteristic flow times. The flow time in a premixed flow is fixed by the burning velocity, which is an eigenvalue of the problem. Hence, diffusion flames are easier to control and more stable since they allow to choose the Damköhler number.

$$Da = \frac{B\rho_{st}\nu'_{O_2}Y_{F,1}}{\chi_{st}W_F\left(1-Z_{st}\right)}e^{\frac{-Ea}{RT}}$$

## Laminar Diffusion Flames

### Counter-Flow Flames

With $t_c = \dfrac{Z_{st}^2 (1 - Z_{st})^2}{\chi_q}$ and $\chi = \dfrac{a}{\pi} e\left(-2[erfc^{-1}(2Z)]^2\right)$ time scales for diffusion flames become available:

$$erfc^{-1}(2Z_{st}) = \begin{cases} 1.34 & \text{for hydrogen-air flames with } Z_{st}=0.0284 \\ 1.13 & \text{for methane-air flames with } Z_{st} = 0.055 \end{cases}$$

Extinction occurs at strain rates $a_q$ of $\begin{cases} 14260/s & \text{for hydrogen-air} \\ 420/s & \text{for methane-air} \end{cases}$

→ $t_c = \begin{cases} 0.64 \cdot 10^{-5} & \text{for hydrogen-air} \\ 0.29 \cdot 10^{-3} & \text{for methane-air} \end{cases}$

# Non-Premixed Flames

## Turbulent Diffusion Flames

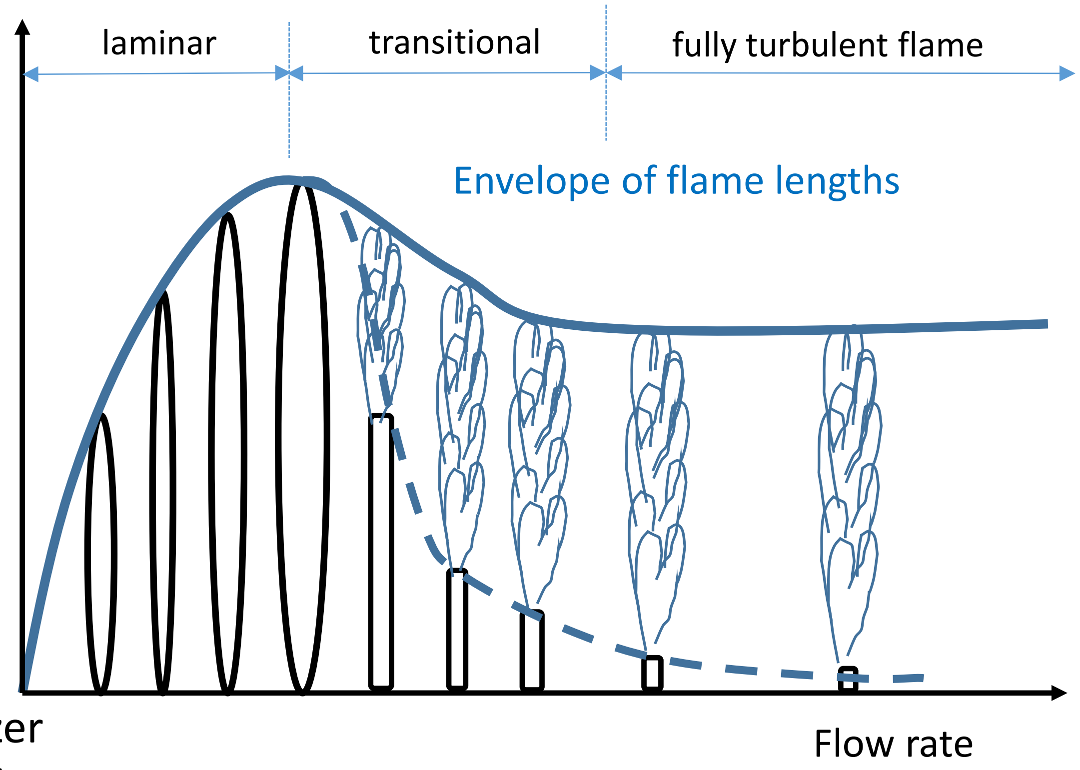
### Circular Jet Flame

Depending on the injection velocity, the flame stays laminar and flame length increases with injection velocity.
Due to increase mixing when jet becomes transitional, the flame length decreases drastically while for fully developed turbulent jets the flame length stays constant until flame lift-off conditions are reached.

Empirical Correlation for flame length:

$$L_f = \frac{23 d_i \left( \frac{\rho_f}{\rho_o} \right)^{1/2}}{Y_{f,st}}$$

$L_f$: flame length
$d_i$: fuel injection diameter
$\rho_f/\rho_o$: density ratio of fuel to oxidizer
$Y_{f,st}$: stoichiometric fuel mass fraction



# Non-Premixed Flames

## Turbulent Diffusion Flames

### Circular Jet Flame

### Assumptions:

Stationary, axisymmetric boundary layer flow without buoyancy, negligible molecular transport, $Sc_t = \nu_t/D_t$, Favre-averaging

<u>Continuity</u>

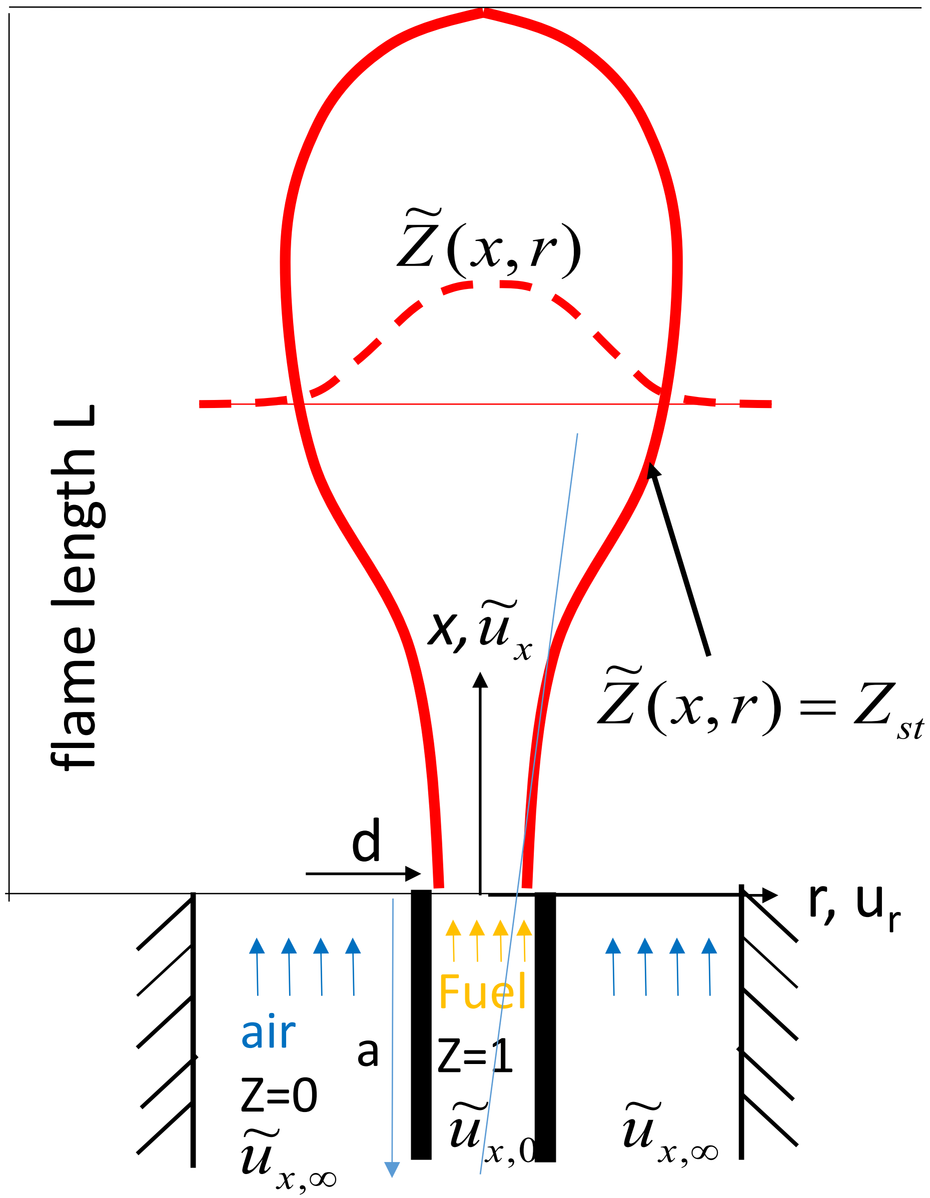
$$\frac{\partial(\bar{\rho}\tilde{u}_x r)}{\partial x} + \frac{\partial(\bar{\rho}\tilde{u}_r r)}{\partial r} = 0$$

<u>Momentum in x-direction</u>

$$\bar{\rho}\tilde{u}_x r \frac{\partial \tilde{u}_x}{\partial x} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{u}_x}{\partial r} = \frac{\partial}{\partial r}\left(\bar{\rho}\nu_t r \frac{\partial \tilde{u}_x}{\partial r}\right)$$

<u>Mean mixture fraction</u>

$$\bar{\rho}\tilde{u}_x r \frac{\partial \tilde{Z}}{\partial x} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{Z}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\bar{\rho}\nu_t}{Sc_t} r \frac{\partial \tilde{Z}}{\partial r}\right)$$



# Turbulent Diffusion Flames

## Circular Jet Flame

### Laminar

$$\frac{\partial(\rho u_x r)}{\partial x} + \frac{\partial(\rho u_r r)}{\partial r} = 0$$

$$\rho u_x r \frac{\partial u_x}{\partial x} + \rho u_r r \frac{\partial u_x}{\partial r} = -r\frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_x}{\partial r}\right)$$

$$\rho u_x r \frac{\partial Z}{\partial x} + \rho u_r r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\mu}{Sc} r \frac{\partial Z}{\partial r}\right)$$

### Turbulent

$\nu_t$ requires solution of $k$ and $\varepsilon$ equations

$$\frac{\partial(\bar{\rho}\tilde{u}_x r)}{\partial x} + \frac{\partial(\bar{\rho}\tilde{u}_r r)}{\partial r} = 0$$

$$\bar{\rho}\tilde{u}_x r \frac{\partial \tilde{u}_x}{\partial x} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{u}_x}{\partial r} = \frac{\partial}{\partial r}\left(\bar{\rho}\nu_t r \frac{\partial \tilde{u}_x}{\partial r}\right)$$

$$\bar{\rho}\tilde{u}_x r \frac{\partial \tilde{Z}}{\partial x} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{Z}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\bar{\rho}\nu_t}{Sc_t} r \frac{\partial \tilde{Z}}{\partial r}\right)$$

# Non-Premixed Flames

## Turbulent Diffusion Flames

### Circular Jet Flame

Jet in quiescent air with a treatment of turbulent equation similar to the laminar case

Laminar

$$\eta = \frac{\sqrt{2\int_0^r \frac{\rho}{\rho_0} r\,dr}}{\phi} \qquad C = \frac{\rho \mu r^2}{2\mu_\infty \int_0^r \rho r\,dr}$$

for $C \approx const.$

$$u_x = \frac{2C\gamma^2 \nu}{\phi\left(1 + (\gamma\eta)^2/4\right)^2};$$

$$\Rightarrow \gamma^2 = \frac{3}{64}\frac{\rho_0}{\rho_\infty}\frac{u_{x,0} d}{\upsilon_\infty}\frac{1}{C^2};$$

Turbulent

$$\eta = \frac{\sqrt{2\int_0^r \frac{\bar{\rho}}{\rho_0} r\,dr}}{\phi} \qquad C_t = \frac{\bar{\rho}^2 \nu_t r^2}{2\rho_\infty \nu_{t,ref} \int_0^r \bar{\rho} r\,dr}$$

for $C_t \approx const.$

$$u_x = \frac{2C_t\gamma^2 \nu_{t,ref}}{\phi\left(1 + (\gamma\eta)^2/4\right)^2};$$

$$\Rightarrow \gamma^2 = \frac{3}{64}\frac{\rho_0}{\rho_\infty C_t^2}\left(\frac{u_{x,0} d}{\nu_{t,ref}}\right)^2$$

# Things you shouldn't forget

- Counter-flow Diffusion Flames
- Turbulent Diffusion Flames

# Annex

# Non-Premixed Flames

## Laminar Diffusion Flames

### Spherical Jet Flame

### Boundary conditions

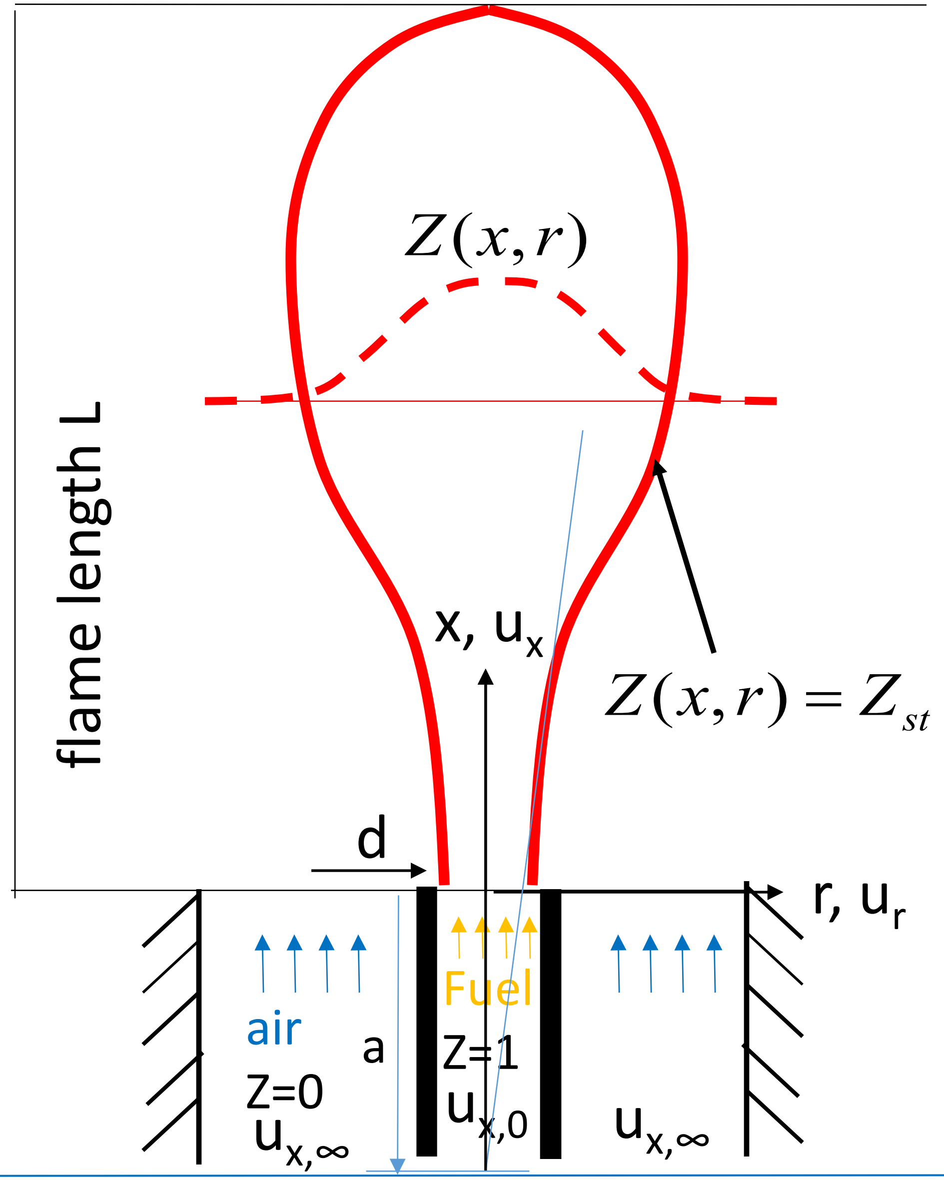
$$r \rightarrow \infty : u_x = u_r = 0; dp/dx = 0;$$

similarity coordinate $\eta = r/x$

For situation where density isn't constant, a transformation is helpful (a: distance of virtual origin of jet from nozzle exit)

$$\phi = x + a; \quad \eta = \frac{\sqrt{2\int_0^r \frac{\rho}{\rho_0} r dr}}{\phi}$$

for $\rho = const.$ and a → 0 $\quad \phi = x; \quad \eta = \frac{r}{x}$



# Non-Premixed Flames

## Laminar Diffusion Flames

### Spherical Jet Flame

### Stream function $\Psi$

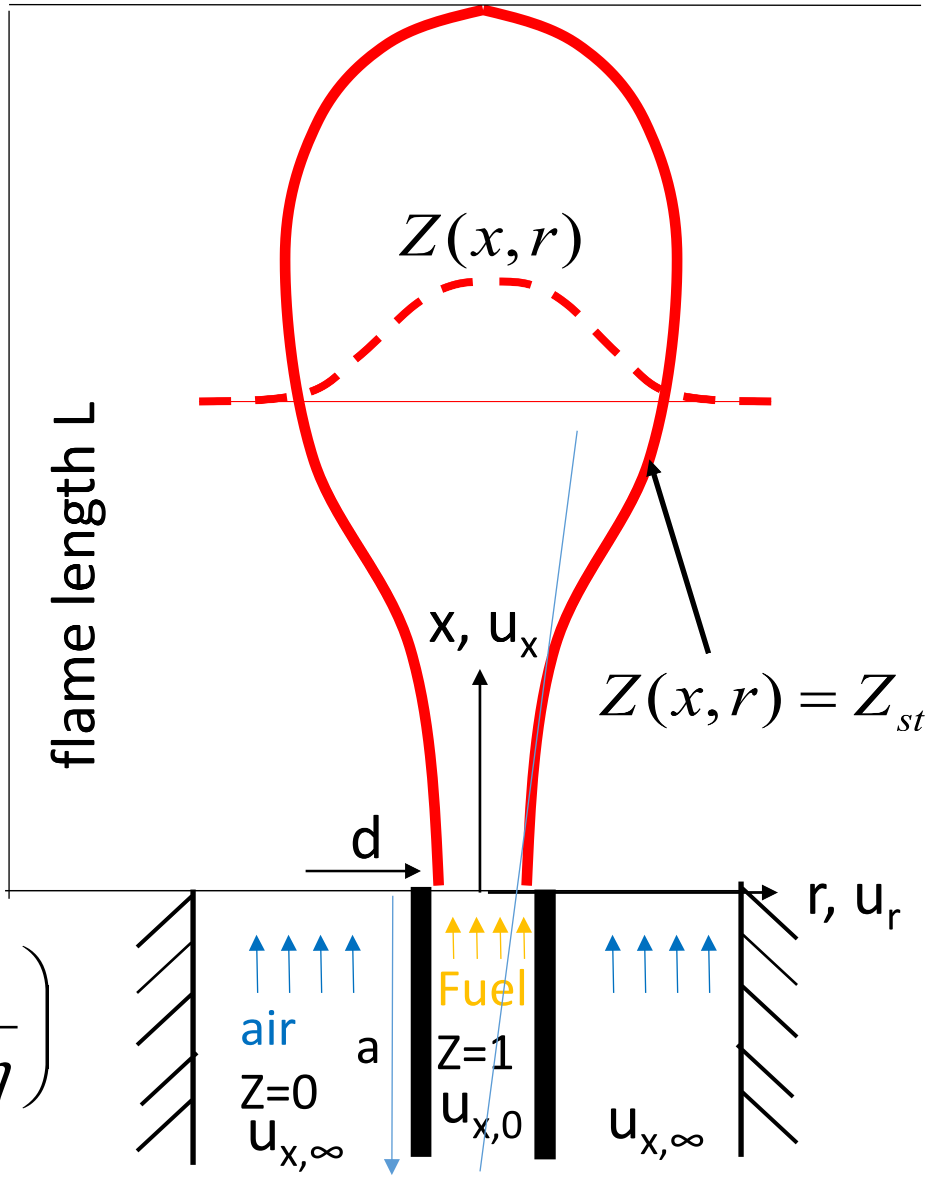
$$\rho u_x r = \frac{\partial \Psi}{\partial r}; \quad \rho u_r r = \frac{\partial \Psi}{\partial x}$$

Apply transformation rules $\phi = x + a; \quad \eta = \frac{\sqrt{2\int_0^r \frac{\rho}{\rho_0} r dr}}{\phi}$

$$\frac{\partial}{\partial x} = \frac{\partial \eta}{\partial \phi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta}; \quad \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta}$$

to convective terms in momentum and mixture fraction equations

$$\rho u_x r \frac{\partial}{\partial x} + \rho u_r r \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\left(\frac{\partial \phi}{\partial \eta}\frac{\partial}{\partial \Psi} - \frac{\partial \Psi}{\partial \phi}\frac{\partial}{\partial \eta}\right)$$



# Non-Premixed Flames

## Laminar Diffusion Flames

### Spherical Jet Flame

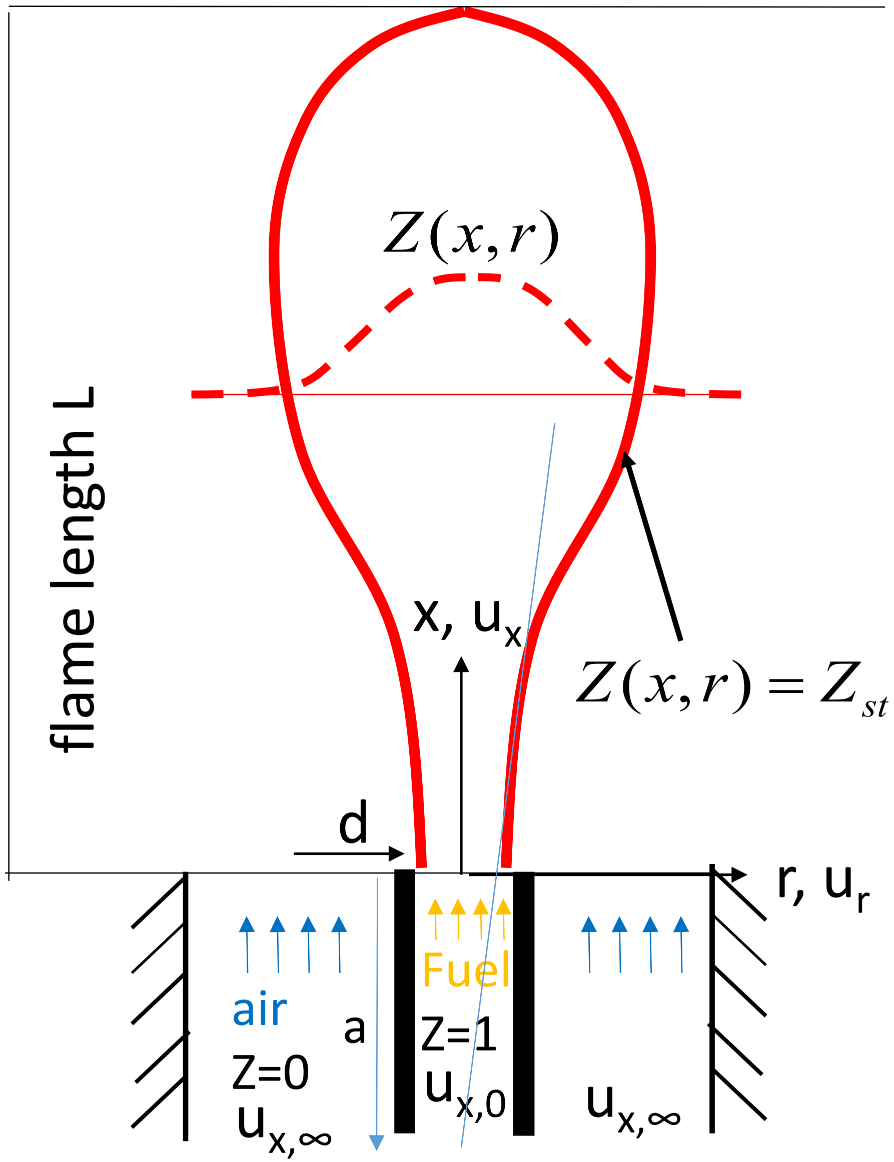
the diffusive terms become
$$\frac{\partial}{\partial x}\left(\mu r \frac{\partial}{\partial r}\right) = \mu_\infty \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta}\left(C\eta \frac{\partial}{\partial \eta}\right)$$

*C*: Chapman-Rubesin parameter
$$C = \frac{\rho \mu r^2}{2\mu_\infty \int_0^r \rho r dr}$$

For constant density (with $\eta = \frac{r}{x}$ and $\mu = \mu_\infty$): *C=1*

Assumption: $C = f(\phi,\eta);\quad \Psi = \eta_\infty \phi F(\phi,\eta);$

$$u_x = \frac{\partial F}{\partial \eta}\frac{1}{\eta}\frac{\mu_\infty}{\rho_\infty \phi};\quad u_r = -\frac{\mu_\infty}{r\rho}\left(\frac{\partial F}{\partial \phi} + F - \eta\frac{\partial F}{\partial \eta}\right);$$



# Non-Premixed Flames

## Laminar Diffusion Flames

### Spherical Jet Flame

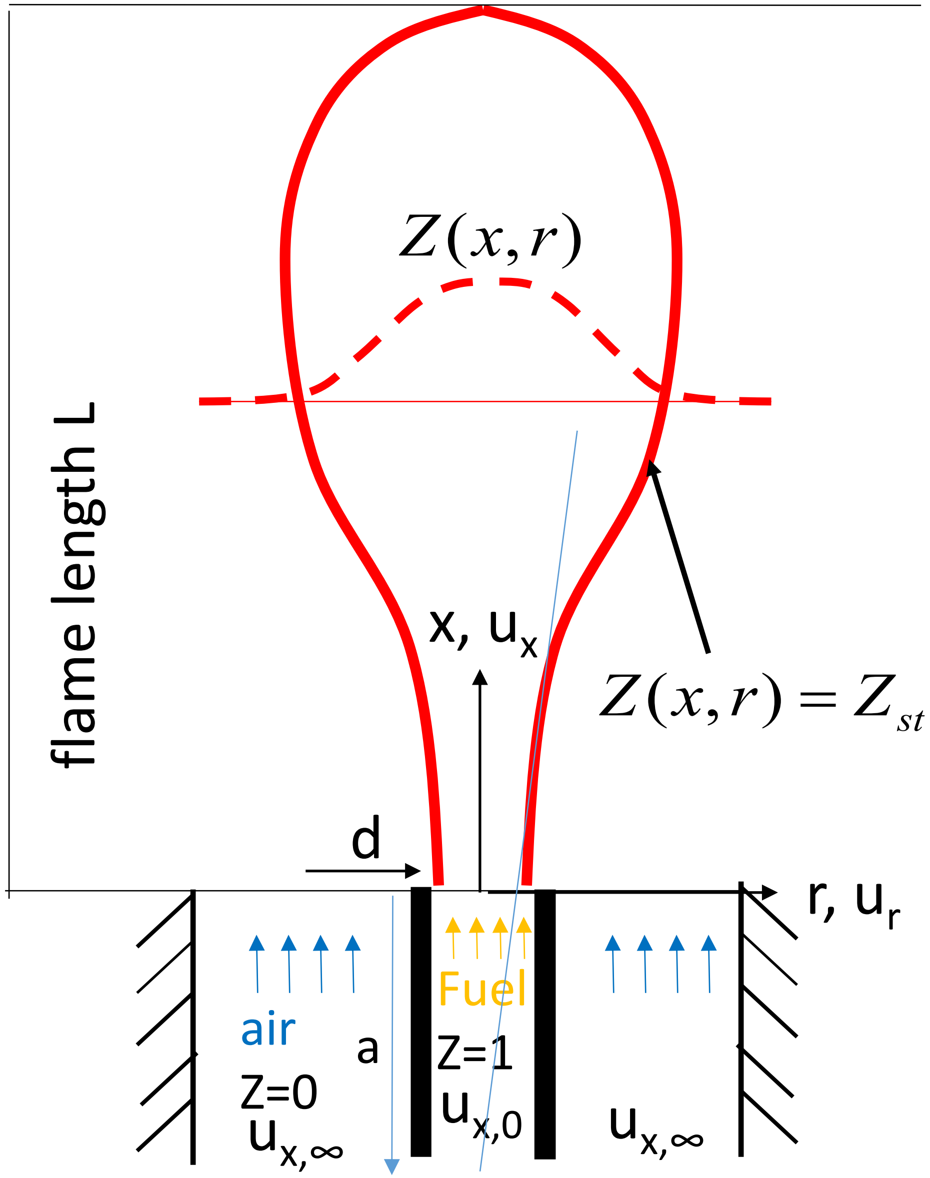
If *C = const.* $F = \frac{C(\gamma\eta)^2}{1+(\gamma\eta)^2/4}; \omega = \left(\frac{1}{1+(\gamma\eta)^2/4}\right)^{2Sc}$

*C = const.* holds if $C = \frac{\rho\mu r^2}{2\mu_\infty \int_0^r \rho r dr 1}; \Rightarrow C = \frac{\rho\mu}{\rho_m \mu_\infty} = const.$

and this is hardly fulfilled since $\mu \sim T^{0.7}$ *and* $\rho \sim T^{-1}$

Integration constant *γ* to be derived from condition that jet momentum independent of *Φ*

$$u_x = \frac{2C\gamma^2\nu}{\phi\left(1+(\gamma\eta)^2/4\right)^2}; \Rightarrow \gamma^2 = \frac{3}{64}\frac{\rho_0}{\rho_\infty}\frac{u_{x,0}d}{\upsilon_\infty}\frac{1}{C^2};$$



# Laminar Diffusion Flames

## Spherical Jet Flame

then the momentum equation $$\rho r u_x \frac{\partial u_x}{\partial x} + \rho r u_r \frac{\partial u_x}{\partial r} = \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_x}{\partial r}\right);$$

yields $$\phi\left(\frac{\partial F}{\partial \eta}\frac{1}{\eta}\frac{\partial}{\partial \phi}\frac{\partial F}{\partial \eta} - \frac{\partial F}{\partial \phi}\frac{\partial}{\partial \eta}\frac{\partial F}{\partial \eta}\frac{1}{\eta}\right) - \frac{\partial}{\partial \eta}\left(F\frac{\partial F}{\partial \eta}\frac{1}{\eta}\right) = \frac{\partial}{\partial \eta}\left(C\eta\frac{\partial}{\partial \eta}\frac{\partial F}{\partial \eta}\frac{1}{\eta}\right)$$

a similarity solution exists only if $F \neq f(\phi);$

and then $u_x = \propto \frac{1}{\phi}; \Rightarrow$ linear velocity decrease with 1/x+a

Recall that: $u_x = 0$ and $u_x = 0$ for $\eta \rightarrow \infty$

# Non-Premixed Flames

## Laminar Diffusion Flames

### Spherical Jet Flame

for the nondimensional stream function

$$-\frac{\partial}{\partial \eta}\left( F \frac{\partial F}{\partial \eta} \frac{1}{\eta} \right) = \frac{\partial}{\partial \eta}\left( C \eta F \frac{\partial}{\partial \eta} \frac{\partial F}{\partial \eta} \frac{1}{\eta} \right)$$

and with

$$\omega = \frac{Z(x,r)}{Z_a(x)}; Z_a(x) = Z(x, r=0);$$

and transformation to $\omega$ – equation

$$\phi\left( \frac{\partial F}{\partial \eta} \frac{\partial \omega}{\partial \phi} - \frac{\partial F}{\partial \phi} \frac{\partial \omega}{\partial \eta} \right) + \phi \frac{\partial F}{\partial \eta} \omega \frac{\partial \ln(Z_a)}{\partial \phi} - F \frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta}\left( \frac{C}{Sc} \eta \frac{\partial \omega}{\partial \eta} \right)$$

similarity →

$$-F \frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta}\left( \frac{C}{Sc} \eta \frac{\partial \omega}{\partial \eta} \right)$$