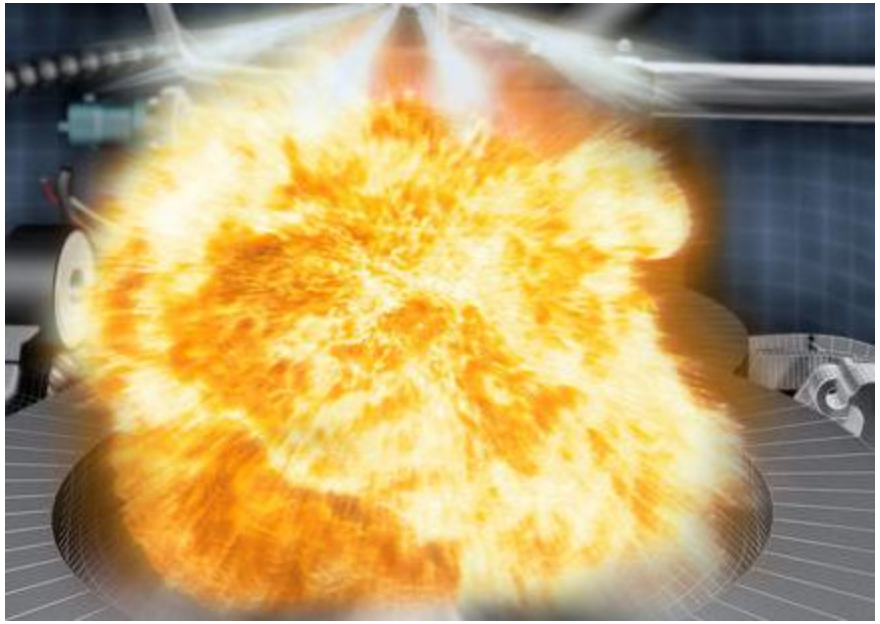
Aerospace Combustion

Lecture 12:

Spray Combustion Part 1



Kerosene Spray Combustion
(AMS kfztech.de)

Spray Combustion

Content

- Injectors and Injection Conditions
- Atomization and Droplet Formation and Break-up
- Droplet Transport
- Vaporization
- Description of 2-Phase Flows
- Droplet Combustion
- Spray Combustion

Injectors and Injection Conditions

Injectors

Injectors for air-breathing engines commonly applying various forms of destabilizing mechanisms of the liquid fuel

- Generation of swirling flows of air and fuel
- Generation of fuel jets or films and break—up through high-speed co-flowing air (pilot or main injector)

Injectors for Rocket Engines are chosen depending on the thermodynamic state of the propellants when they enter the combustion chamber as well as on the mission of the engine

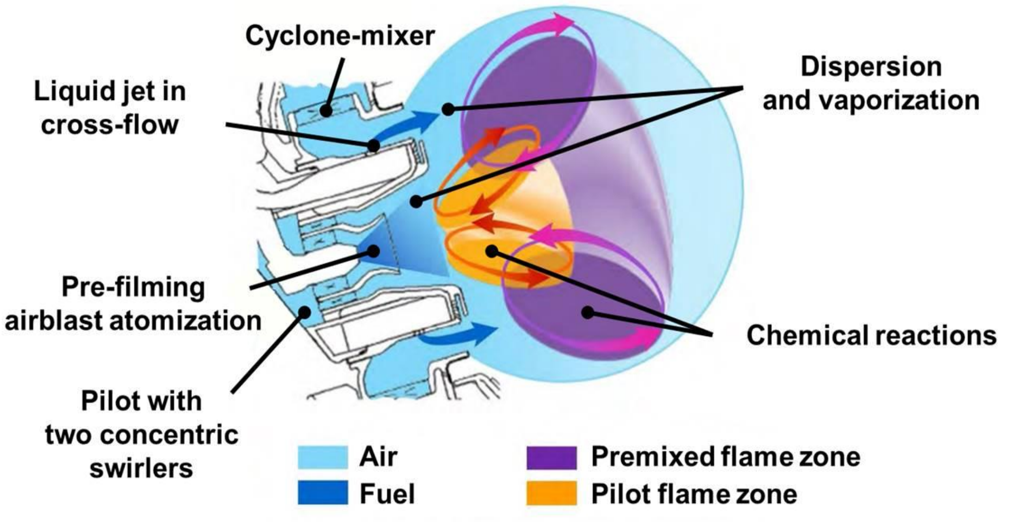
- Shear coax injectors w/wo swirl for gas/liquid propellants and sometimes as well for thrust variation applications
- Variations of impinging jets for liquid/liquid mixtures w/wo thrust regulation requirement

Injectors and Injection Conditions

Injectors for Aero-engines

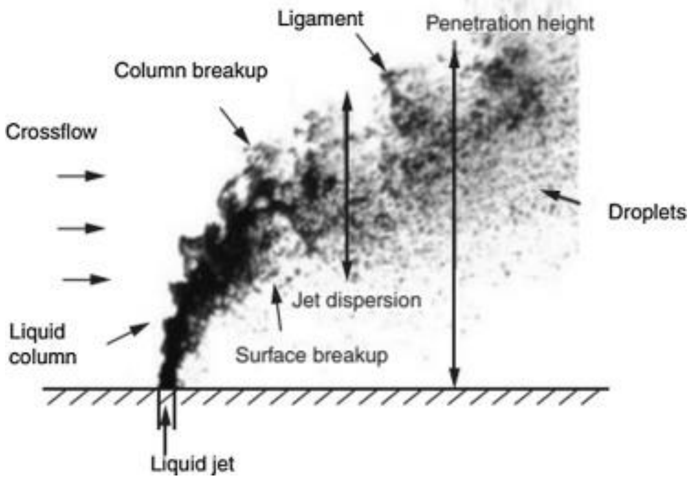
Two types of injectors which apply different physical break-up phenomena

- Jet in cross-flow
- Air-blast atomizer w/wo pre-filming

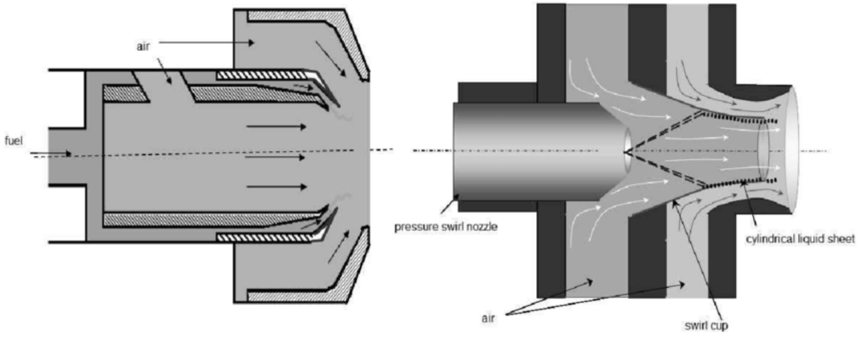


GE Twin Annular Premixing Swirler (TAPS) combustion system

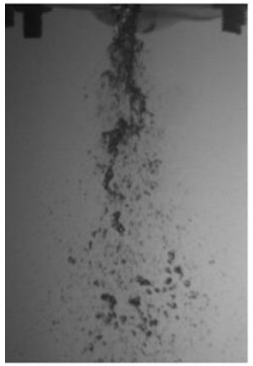
from Eckel 2018 (adapted from Foust et al. 2012, Stickles & Barrett 2013)



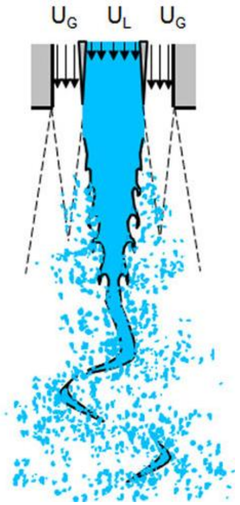
Jet in cross-flow



Air-blast w/wo pre-filming



Air-blast

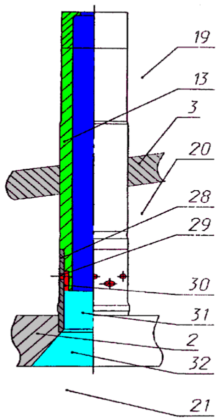


Injectors and Injection Conditions

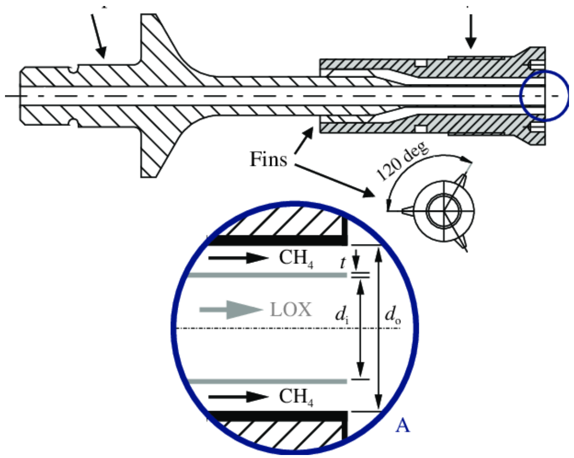
Injectors for Rocket Engine

Two classes of injectors depending on state of propellants

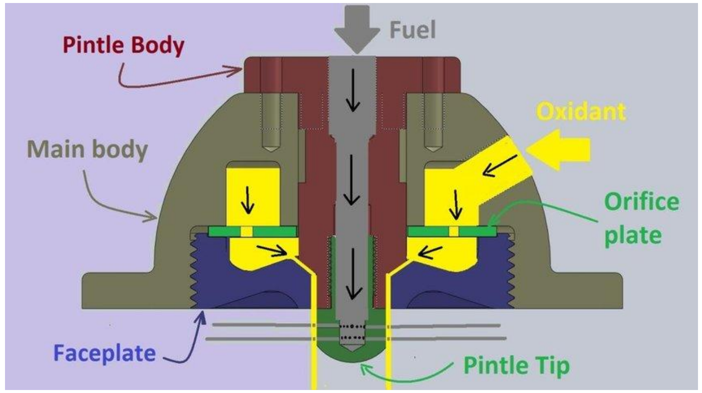
- Gas/liquid injectors w/wo swirl
  - Shear coax
  - Swirl coax
- Liquid/liquid injectors
  - Impinging jets
  - Pintle type

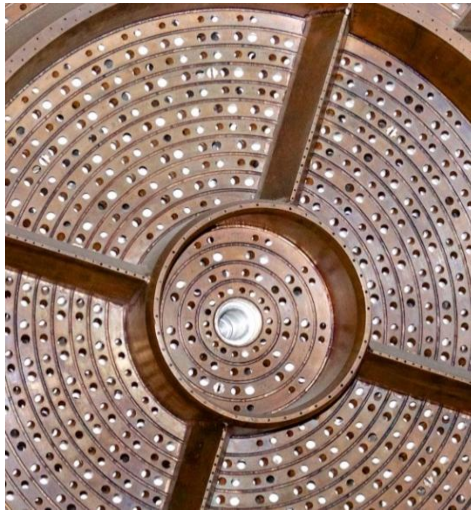


Swirl coax injector

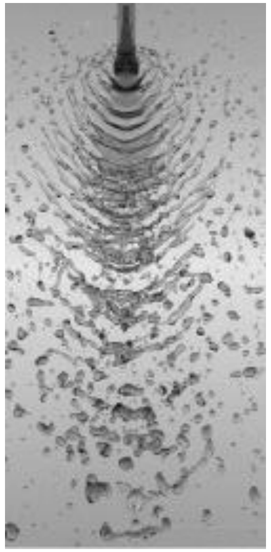


Shear-coax injector



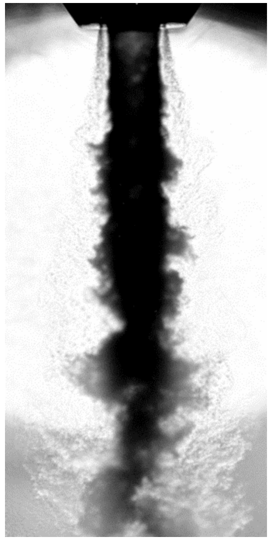


Face plate of F-1 engine with like-on-like impinging injection



Spray pattern of a doublet impinging injector

Spray pattern of a doublet impinging injector



Injectors and Injection Conditions

Injection Conditions

Both in Aeronautics and space, propellants are injected as sub-, or trans-critical fluids. While in aero-engines this is typically kerosene, in rocket engines the propellants are either both in liquid state or at least one of it. Even in full flow cycle engines the pre-burners will be operated with at least one being liquid.

Aero Engines

Engines with high pressure ratios (> 30:1) engines have trans-critical injection conditions

Rocket Engines

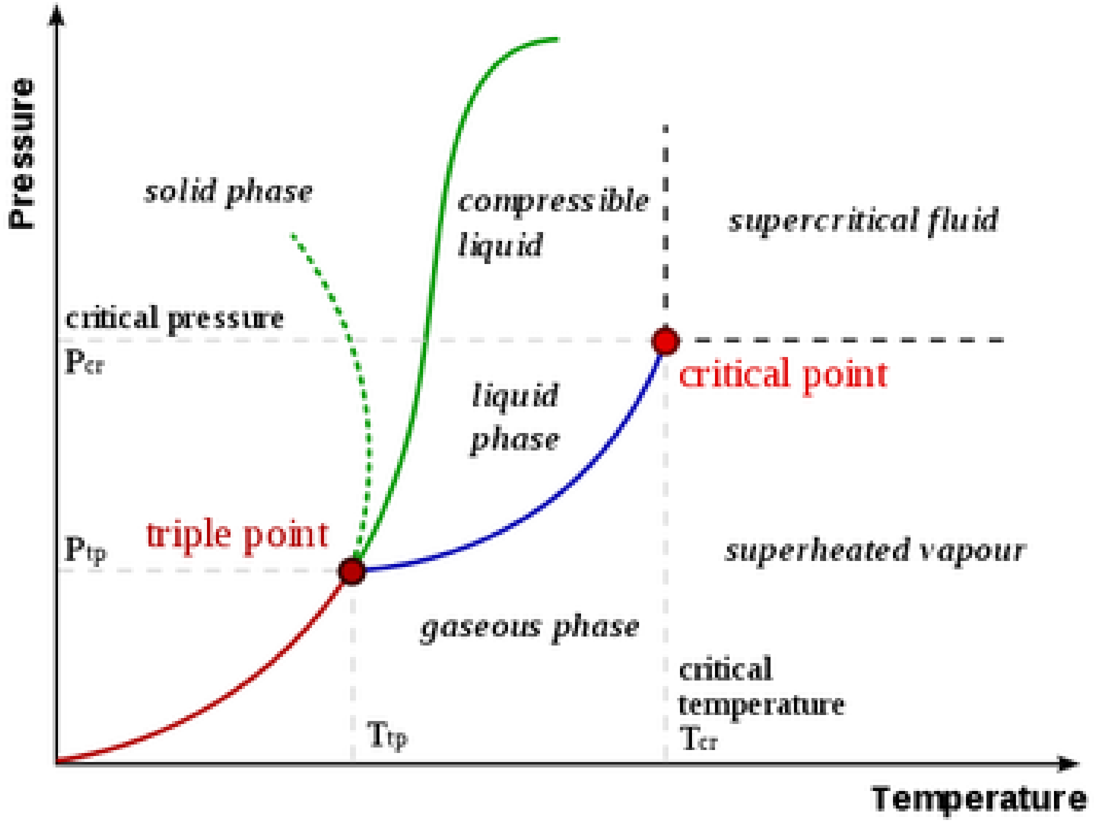
Application- and propellant-specific injection conditions

- Storable propellants are sub-, trans or supercritical
- Kerosene and Oxygen both trans-critical
- Cryogenic fuels are injected trans- or supercritical

Injectors and Injection Conditions

Thermodynamics of Liquid Propellants

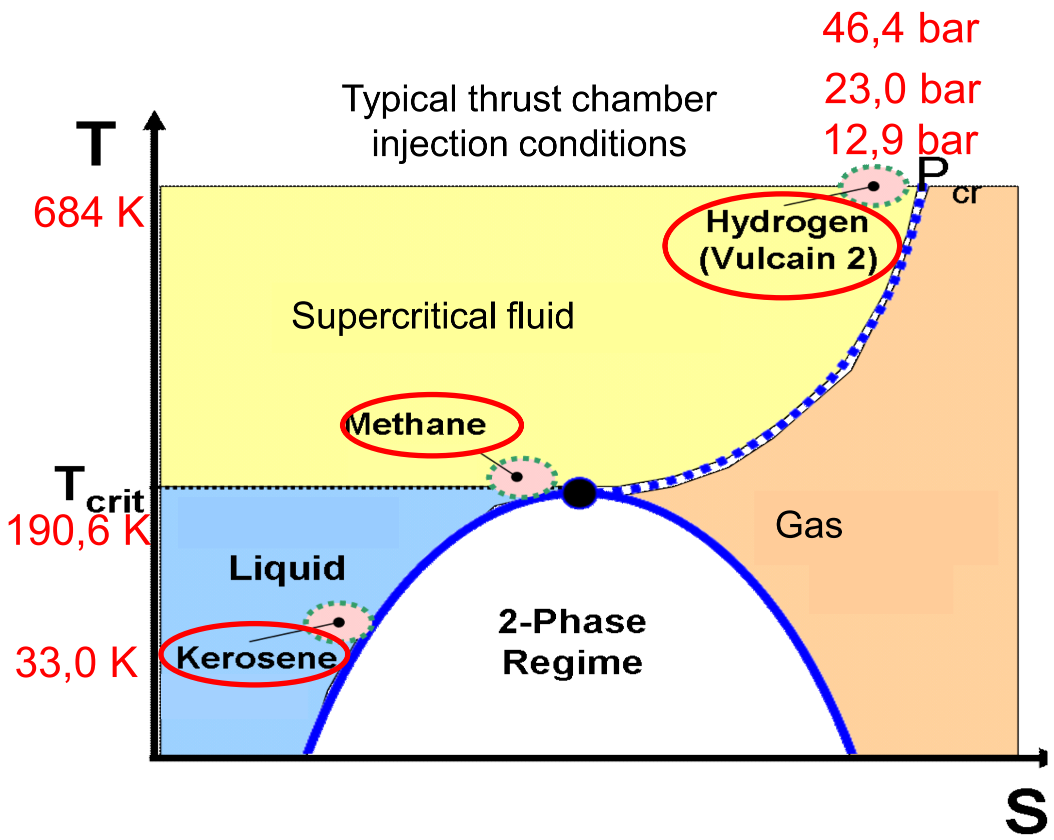
- Critical point at end of vapor pressure curve
- Densities of liquid and gaseous phase become identical
- Critical temperature indicates temperature below which a gas can be in liquid state
- Trans-critical state describes a fluid which is above its critical pressure but below its critical temperature
- Super-critical describes a state where both pressure and temperature are above their critical values



Injectors and Injection Conditions

Thermodynamics of Liquid Propellants

- At near-critical conditions the latent heat vanishes and almost all thermo-physical properties undergo significant changes
- For super-critical conditions surface tension has vanished (important for injector design)
- For regenerative cooling cycles critical conditions are essential for efficiency and performance
- Operation at super-critical pressures avoids phase change phenomena (competition between very efficient bubble boiling and the poor efficiency film boiling)



Atomization and Droplet Formation

Atomization

Liquid propellants are favored to allow for smaller tanks and reduced weight

However, combustion is mostly a gas phase process

↓

Disintegration and Vaporization Required

Depending on the injector design, the dominating physical process will be different

- Swirl injectors generate a liquid film which disintegrates into smaller droplets and ligaments typically applied in gas turbines and some rocket engines
- Shear coax injectors rely on the destabilizing aerodynamic forces to break-up the liquid jet and larger fluid elements and generally produces smaller droplets than droplets
- Impinging and pintle injectors generate an unstable liquid sheet which disintegrates into droplets

Atomization and Droplet Formation

Atomization

General Statements

- Turbulent 2-phase flows are intrinsically unstable
- Degree of liquid jet or film disintegration depends on acting forces both inside the liquid as well at its surface;
- Low velocity differences -> Rayleigh instability
- High velocity differences -> Kelvin-Helmholtz instability

Problem split in two Break-up Regimes

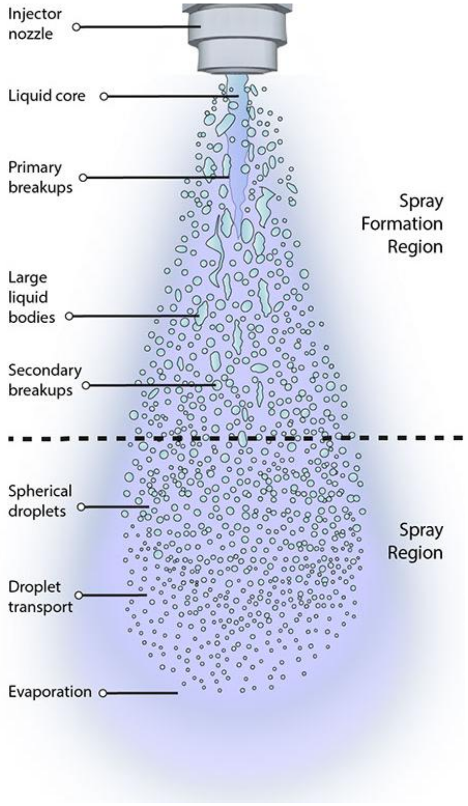
Primary Break-up:

- Droplets ejected from the liquid surface and formation of ligaments

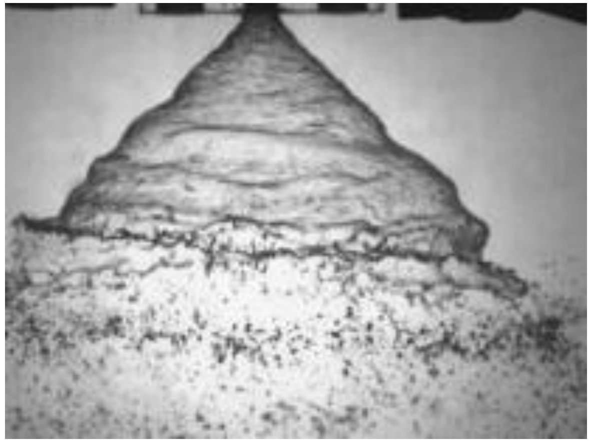
Secondary Break-up:

- Break-up of ligaments and larger droplets into small droplets

Liquid jet break-up regimes:
Primary break-up of small droplets and formation of ligaments
Secondary break-up of ligaments and droplets



Swirl Atomizer break-up regimes:
Formation of film, break-up of film into primary droplets and ligaments
Secondary break-up of ligaments and droplets



Atomization and Droplet Formation

Atomization

Non-dimensional numbers characteristic for atomization (Reynolds, Weber, Momentum Flux Ratio, Ohnesorge, Density Ratio)

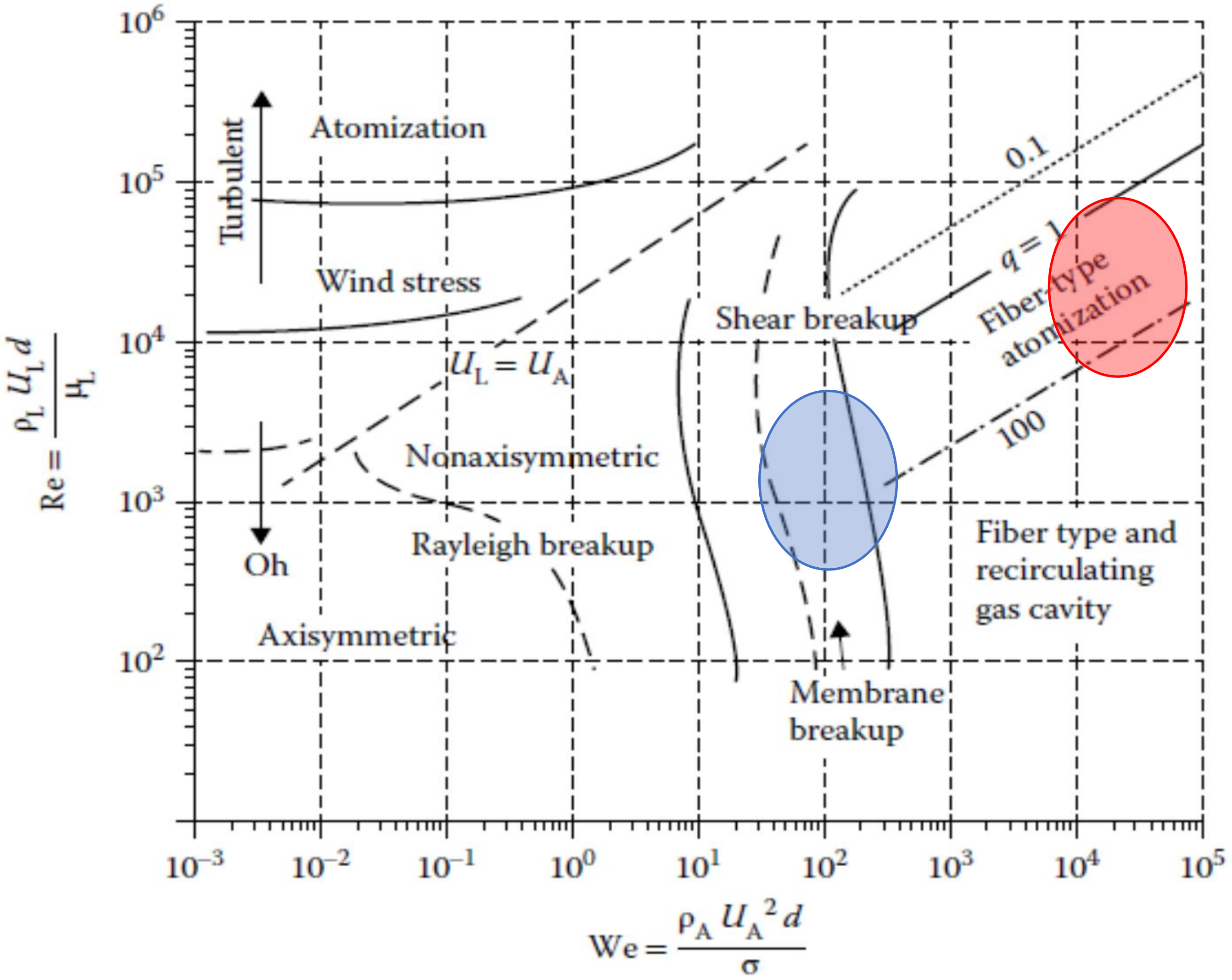
$$Re = \frac{u \mu d}{\rho}; \; We = \frac{\rho_g u_g^2 d_l}{\sigma}; \; M = \frac{\rho_g u_g^2}{\rho_l u_l^2} = q;$$

$$Oh = \frac{\eta_l}{\sqrt{d_l \rho_l \sigma}} = \frac{\sqrt{We}}{Re}; \quad DR = \frac{\rho_l}{\rho_g}$$

Injectors for rocket engines generally work in the region of very high Weber and Reynolds numbers

Injectors for aero-engines are characterized by much smaller Reynolds and Weber numbers

Remark: Even though for a large number of rocket engines injection pressures are above the critical pressure where surface tension isn't defined anymore, the Weber number is still used to describe injection conditions.



© Lasheras, Hopfinger, Annual Review of Fluid Mechanics, 2000

Droplet Formation

Liquid propellants are favored to allow for smaller tanks and reduced weight. However, combustion is mostly a gas phase process.

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Disintegration and Vaporization Required

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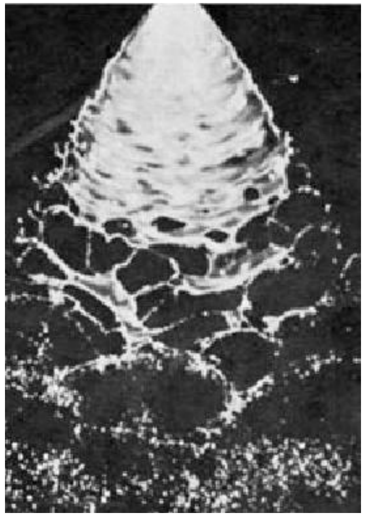
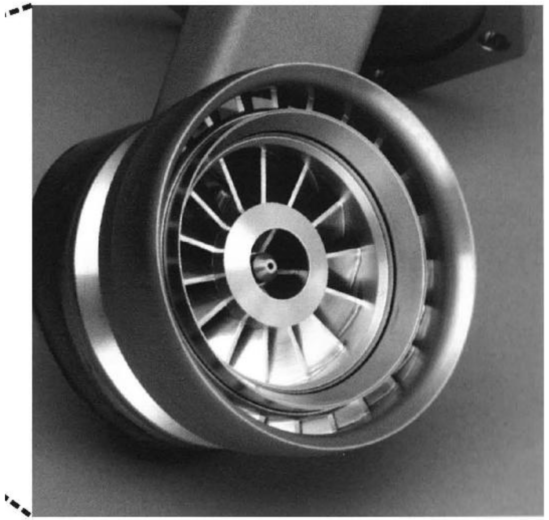
- Impinging and pintle injectors generate an unstable liquid sheet which disintegrates into droplets

Droplet Formation

Disintegration of two different initial configurations

- Sheets (impinging, swirl)
  - Formation of waves
  - Sheet disintegration and formation of ligaments and primary droplets at the rims of the sheet
  - Disintegration of ligaments and secondary droplet break-up
- Jets (shear coax, jet in cross-flow)
  - Primary droplet formation
  - Jet disintegration and formation of ligaments
  - Disintegration of ligaments and secondary droplet break-up





Temporal and Spatial Variation of Droplet Size and Droplet Number Density

Atomization and Droplet Formation

Particle Size Distribution

Particles generated by milling, crushing or atomization can fairly well be described by a 2 parameter Weibull distribution of the type

$$f(x,\lambda,k)=\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k}$$

with *x,* the diameter, and the shape and scale parameters λ and *k > 0*

The cumulative distribution function is then

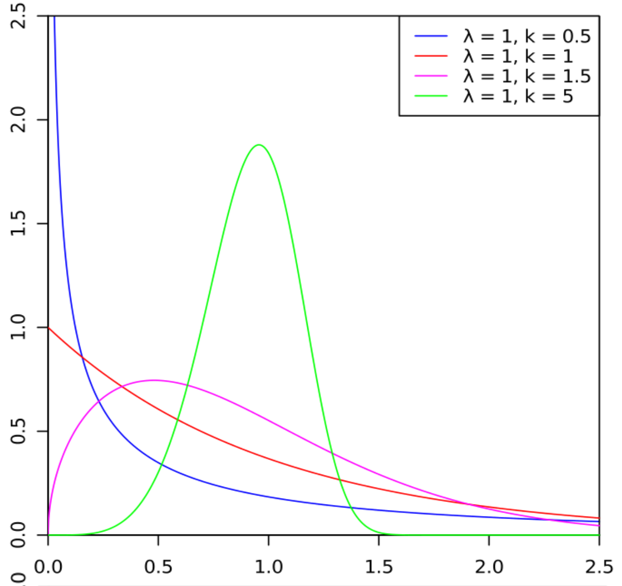
$$P(x,\lambda,k)=1-e^{-(x/\lambda)^k}$$

the mean, $$\bar{x}=\lambda\Gamma(1+1/k)$$

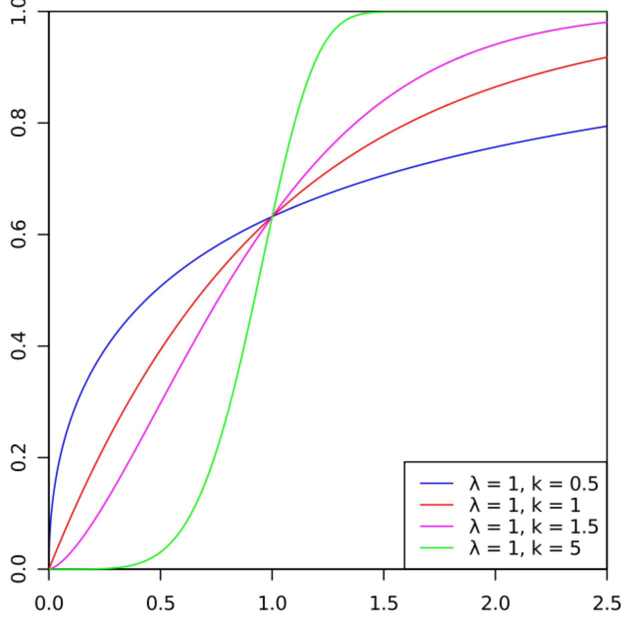
and the variance $$\sigma^2=\lambda^2\left[\Gamma(1+2/k)-(\Gamma(1+1/k))^2\right]$$

Rosin and Rammler were the first to use this function to describe particle size distributions.

Probability density distribution function



Cumulative distribution function



Atomization and Droplet Formation

Sheet Formation (Pressure Swirl Atomizer)

Film centrifugal motion generates a gaseous core. Film thickness *t* is related to the mass flow rate and the nozzle diameter *d* by

$$\dot{m} = \pi \rho_l u t (d - t);$$

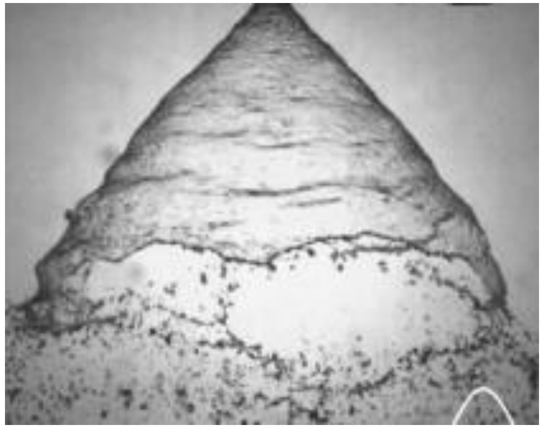
With *u* the axial velocity of the liquid at nozzle exit which is related to the spray angle ϑ. The total velocity *U* is assumed to be related to the pressure loss *Δp* by

$$U = k_v \sqrt{\frac{2\Delta p}{\rho_l}}; \quad u = U \cos\theta \qquad \text{with} \quad k_v = \max\left| 0.7; \frac{4\dot{m}}{d^2 \rho_l \cos\theta} \sqrt{\frac{\rho_l}{2\Delta p}} \right|$$

Assuming a 2D sheet of thickness 2h moving with velocity U and using a coordinate system which moves with the sheet and a spectrum of small disturbance of the form imposed on the initially steady motion

$$\eta = \eta_0 e^{ikx + \omega t};$$

With η_0 the initial wave amplitude, *k* the wave number and $\omega = \omega_r + i\omega_i$ the complex growth rate. The most unstable disturbance has the largest value of the real part ω_r and thus a dispersion relation for ω as a function of k is desired.



The velocity coefficient k_v is a function of injector design and injection pressure and has to be smaller than unity.

Atomization and Droplet Formation

Sheet Formation and Break-up (Pressure Swirl Atomizer)

There are two modes, the sinuous and the varicose one with the first being dominant. Hence, the dispersion relation for this mode is given by

$$\omega^2\left[\tanh(kh)+Q\right]+\left[4\upsilon_l k^2\tanh(kh)+2iQkU\right]+4\upsilon_l k^4\tanh-4\upsilon_l^2k^3l\tanh(lh)-QU^2k^2+\frac{\sigma k^3}{\rho_l};$$

with $Q=\frac{\rho_g}{\rho_l}$; $l^2=k^2+\frac{\omega}{\upsilon_l}$

An order of magnitude analysis for typical applications reveals that second order terms in viscosity may be neglected which leads the real part in the growth rate to be

$$\omega_r=\frac{1}{\left[\tanh(kh)+Q\right]}\left\{-2\upsilon_l k^2\tanh(kh)\right\}+\sqrt{4\upsilon_l^2k^4\tanh^2(kh)-Q^2U^2k^2-\left[\tanh(kh)+Q\right]\left[-QU^2k^2+\frac{\sigma k^3}{\rho_l}\right]};$$

We have to distinguish now between two cases, when the wave length is either long or short compared with the sheet thickness.

Atomization and Droplet Formation

Sheet Formation and Break-up (Pressure Swirl Atomizer)

For long waves, Dombrowski and Johns assumed the sheet would break-up once a critical surface disturbance value η_b was achieved and from this a break-up time τ could be deduced.

$$\eta_b = \eta_0 e^{\Omega\tau} \Rightarrow \tau = \frac{1}{\Omega}\ln\left(\frac{\eta_b}{\eta_0}\right);$$

with Ω, the maximum growth rate to be determined numerically maximizing the real part equation for ω_r as a function of k.

The sheet will break-up at a length L

$$L_b = U\tau = \frac{U}{\Omega}\ln\left(\frac{\eta_b}{\eta_0}\right);$$

the ratio of the disturbances η_b/η_0 ~ 20, is an empirical constant valid for $2 < We < 200$

The sheet diameter d_L can be determined by $d_L = \sqrt{\frac{8h}{K_s}}$

The ligament diameter now depends on the sheet thickness which is a function of the break-up length and the radial distance from the atomizer exit, r_0.

$$h_b = \frac{r_0 h_0}{r_0 + L_b \sin(\theta/2)}$$

Atomization and Droplet Formation

Sheet Formation and Break-up (Pressure Swirl Atomizer)

For short waves, the ligament diameter is assumed to be linearly proportional to the wavelength that breaks up the sheet.

$$d_L = \frac{2\pi\, C_L}{K_s};$$

with C_L, the ligament constant equal to 0.5;

Independent of the long or short wave case, the break-up from ligaments to droplets is assumed to follow Weber's analysis for capillary instability:

$$d_0 = 1.88 d_L \left(1 + Oh\right)^{1/6};\; Oh = \frac{\sqrt{We}}{\mathrm{Re}}$$

Now finally, one may assume that this diameter is the most probable droplet size of a Rosin-Rammler distribution with an empirically determined spread parameter of 3.5. Furthermore, one has to either assume an empirically established spread angle or apply an empirical value.

Atomization and Droplet Formation

Jet Atomization

Along the liquid jet the local boundary conditions are changing and therefore the driving forces for liquid jet break-up. Important are the velocities of liquid and gas and turbulence intensities, the ratios of velocity and density and the surface tension.

Models for droplet sizes are only valid for a single destabilizing mechanism and therefore actual sprays are described with a size distribution of droplets, i.e. Rosin-Rammler or an equivalent diameter, i.e. Sauter mean diameter D_{32} which is the ratio of volume to surface of a spray.

Correlations for the length of the intact core L are commonly used together with a conical shape of the core from which droplets with random size and pre-defined velocity vectors are ejected.

M is defined as the momentum flux ratio between gas and liquid.

$$\frac{L}{d_l} = \frac{1}{2C_1 M^{2/3}} \left(\frac{\sigma}{\mu_l u_g} \right)^{1/3} \text{; adjustable } C_1$$

$$\frac{L}{d_l} = \frac{6}{\sqrt{M}} \left(\left| 1 - \frac{u_l}{u_g} \right| \right)^{-1} \text{; for We} >> 1$$

$$\frac{L}{d_l} = \frac{6}{\sqrt{M}} \left(\frac{1}{1 - B_1 \sigma / \mu_l u_g} \right)^{1/2} ; \quad B_1 \approx 10^{-3}$$

Atomization and Droplet Formation

Jet Break-up and Atomization (Wave Break-up Model)

We don't deal here with the Taylor-Analysis-Breakup (TAB) Model since it is only suited for small Weber numbers (We < 200). Instead, we focus on the Wave Break-up Model (Reitz, Bracco).

Following an analysis similar to sheet break-up discussed previously, we impose a surface displacement of the form

$$\eta = \eta_0 e^{ikz+\omega t}$$

With η_0 the initial infinitesimal wave amplitude, *k* the wave number and again a dispersion relation which links the growth rate ω to the wave number k is desired.

Using linearized equations for the hydrodynamics of the liquid assuming wave solutions of

the form $\phi_1 = C_1 I_0(kr) e^{ikz+\omega t}$ $\quad \psi_1 = C_2 I_1(Lr) e^{ikz+\omega t}$ with Φ_1 and ψ_1 the velocity potential and stream function, C_1 und C_2 integration constants and I_0 and I_1 modified Bessel functions of the first kind, and

$$L^2 = k^2 + \omega / \nu_l$$

Atomization and Droplet Formation

Jet Break-up and Atomization (Wave Break-up Model)

With the liquid pressure comes from the inviscid part of the liquid equations and the inviscid gas equation can be solve to obtain the fluctuating gas pressure at a position *r=a*.

$$-p_{21} = -\rho_g \left(U - i\omega k\right)^2 k\eta \frac{K_0(ka)}{K_1(ka)}$$

K_0 and K_1 modified Bessel functions of the second kind, and *U* is the relative velocity between gas and liquid.

The linearized boundary conditions are mathematical statements of the liquid kinematic free surface condition, continuity of shear stress and normal stress For the velocity component $v_2 = 0$.

$$v_1 = \frac{\partial \eta}{\partial t}; \qquad \frac{\partial u_1}{\partial r} = -\frac{\partial v_1}{\partial z}$$

$$-p_1 + 2\mu_1 - \frac{\sigma}{a^2}\left(\eta + a^2 \frac{\partial^2 \eta}{\partial z^2}\right) + p_2 = 0$$

The first two boundary conditions can be used to eliminate the integration constants.

$$\omega^2 + 2v_1 k^2 \omega \left[\frac{I_1'(ka)}{I_0(ka)} - \frac{2kL}{k^2+L^2}\frac{I_1(ka)}{I_0(ka)}\frac{I_1'(ka)}{I_0(ka)}\right] =$$

Implementation yields the desired dispersion relation.

$$\frac{\sigma k}{\rho_l}\left(1-k^2a^2\right)\left(\frac{L^2-a^2}{L^2+a^2}\right)\frac{I_1(ka)}{I_0(ka)} + \frac{\rho_g}{\rho_l}\left(U - i\frac{\omega}{k}\right)^2 \left(\frac{L^2-a^2}{L^2+a^2}\right)\frac{I_1(ka)}{I_0(ka)}\frac{K_0(ka)}{K_1(ka)}$$

Atomization and Droplet Formation

Jet Break-up and Atomization (Wave Break-up Model)

Following a similar argumentation concerning the maximum growth (most unstable wave) we get for the maximum growth rate Ω and the corresponding wave length Λ

$$\Omega\left(\frac{\rho_l a^3}{\sigma}\right) = \frac{\left(0.34 + 0.38 We_g^{1.5}\right)}{(1+Oh)\left(1+Ta^{0.6}\right)}; \qquad Ta = Oh\sqrt{We_l}; \quad We_g = \frac{\rho_g U^2 a}{\sigma}$$

$$\frac{\Lambda}{a} = 9.02 \frac{\left(1+0.45 Oh^{0.5}\right)\left(1+0.4 Ta^{0.7}\right)}{\left(1+0.87 We_g^{1.67}\right)^{0.6}}$$

Droplet Break-up

Droplet radius of newly formed by break-up of parcels bases on the assumption that it is proportional to the fastest growing unstable surface wave

$$r = B_0 \Lambda; \quad B_0 = 0.61$$

rate of change of droplet radius

$$\frac{da}{dt} = -\frac{(a-r)}{\tau}; \quad r \leq a$$

break-up time

$$\tau = \frac{3.762 B_1 a}{\Lambda \Omega}; \quad B_1 = 1.73 \text{ (however it may vary between 1 and 60)}$$

Atomization and Droplet Formation

Droplet Distribution Functions

Generally, sprays are characterized by logarithmic probability distribution functions of the form

$$\left(D_{jk}\right)^{j-k} = \frac{\int_0^\infty D^j F(D)dD}{\int_0^\infty D^k F(D)dD};$$

j=1,k=0: D_{10}: arithmetic mean diameter

j=3,k=0: D_{30}: volume average diameter

j=3,k=3: D_{32}: Sauter mean diameter

- Normal Distribution Function

$$D_{jk} = D_{30} \exp\left(\frac{k+j-6}{4\delta^2}\right);$$

- Rosin-Rammler

$$\left(D_{jk}\right)^{j-k} = \left(D_{ref}\right)^{j-k} \frac{\Gamma\left(\frac{j-3}{\delta}+1\right)}{\Gamma\left(\frac{k-3}{\delta}+1\right)};$$

- Nukiyama-Tanasawa

$$\left(D_{jk}\right)^{j-k} = b^{-(j-k)/\delta} \frac{\Gamma\left(\frac{j+3}{\delta}\right)}{\Gamma\left(\frac{k+3}{\delta}\right)};$$

Droplets in Gases

Forces on Droplets

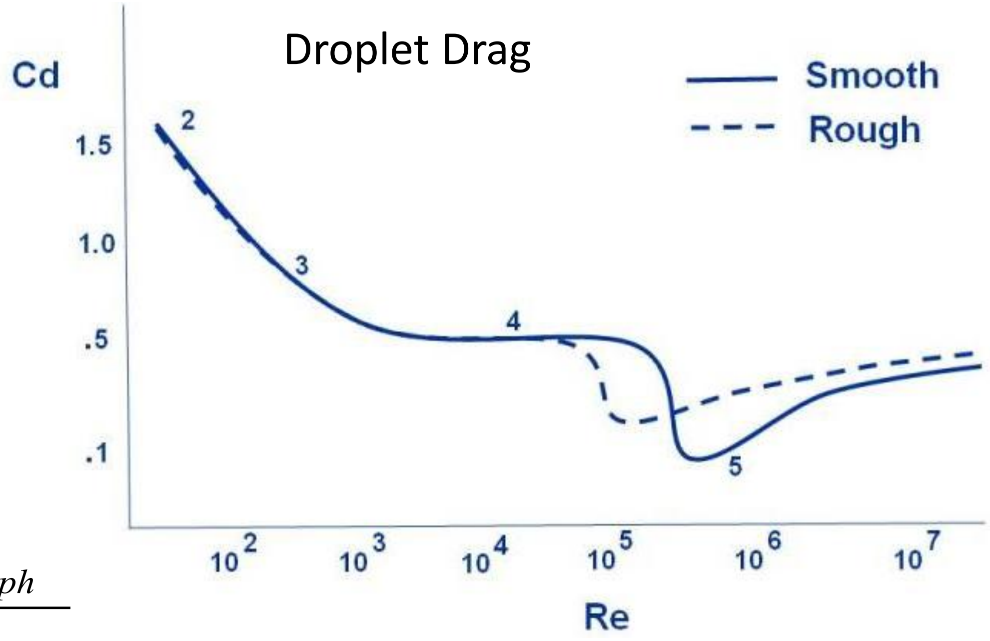
Balance Equation

$$\frac{du_p}{dt} = F_D\left(u_g - u_p\right) + \frac{g_x\left(\rho_p - \rho_g\right)}{\rho_p} + F_x$$

F_x: additional forces acting on droplet

$$F_D = \frac{18\mu}{\rho_p d_p^2}\frac{C_D}{24}; \mathrm{Re} = \frac{\rho_g d_p \left|u_p - u_g\right|}{\mu}$$

C_D: drag coefficient



$$C_D = a_1 + \frac{a_2}{\mathrm{Re}} + \frac{a_3}{\mathrm{Re}^2}; \quad \text{or} \quad C_D = \frac{24}{\mathrm{Re}_{sph}}\left(1 + b_1 \mathrm{Re}_{sph}^{b_2}\right) + \frac{b_3 \mathrm{Re}_{sph}}{b_4 + \mathrm{Re}_{sph}}$$

$$b_1 = \exp\left(2.3288 - 6.4581\phi + 2.4486\phi^2\right)$$

$$b_2 = 0.0964 + 0.5565\phi$$

$$b_3 = \exp\left(4.905 - 13.8944\phi + 18.442\phi^2 - 10.2599\phi^3\right)$$

$$b_4 = \exp\left(1.4681 + 12.254\phi - 20.7322\phi^2 + 15.8855\phi^3\right)$$

$$\phi = \frac{s}{S}$$

Shape factor with *s* surface of sphere and *S* surface of particle

Droplets in Gases

Forces on Droplets

Balance Equation

F_x: additional forces acting on droplet

$$F_{x,1} = \frac{1}{2}\frac{\rho_g}{\rho_p}\frac{d}{dt}\left|u_p - u_g\right|$$

'virtual mass force': force required to accelerate fluid which surrounds the particle

$$F_{x,2} = \frac{\rho_g}{\rho_p} u_p \frac{du_g}{dx}$$

force due to a pressure gradient in the fluid

$$F_{x,3} = D_{T,p}\frac{1}{m_p T}\frac{\partial T}{\partial x}$$

force due to the presence of a temperature field (small particles only)

$$F_{x,4} = \frac{2K\nu^{1/2}\rho_g d_{ij}}{\rho_p d_p \left(d_{lk} d_{kl}\right)^{1/4}}\left(\vec{\mathbf{v}}_g - \vec{\mathbf{v}}_p\right)$$

Saffman lift force: force due to the presence of velocity gradients;
d_{ij}: : deformation tensor
$K = 2.594$

Droplet Transport Modeling

Interface Tracking Methods

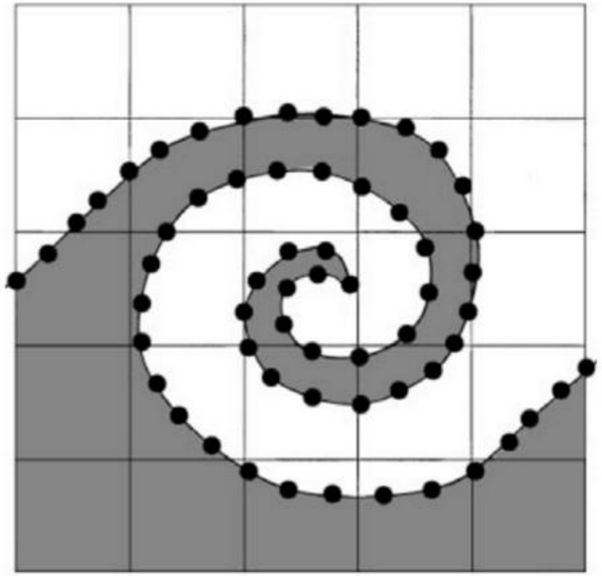
1. Moving Grids
   - ✓ accurate for small deformations
   - very complex in particular for topology changes and normal movements
2. Marker Particles
   - ✓ Accurate
   - Very complex in 3D
   - Topology changes by manual intervention
   - Normal interface movement (mass transfer / phase change) not automatically
3. Volume of Fluid (VOF)
   - ✓ Good volume conservation
   - Interface geometry reconstruction challenging
   - Normal interface movement (mass transfer / phase change) not straightforward
4. Level Sets
   - ✓ Simple interface geometry reconstruction
   - ✓ Normal interface movement automatically
   - Not inherently volume conserving

Droplet Transport Modeling

Interface Tracking Methods

Marker Particles

- Phase tracking by introduction of Lagrangian marker particles in a fixed grid
- Interface reconstruction by polynomials through neighboring marker particles
  - Very accurate interface geometry
  - Requires track keeping of marker connectivity information
  - Difficulties with topology changes
  - Normal interface movement handling (phase change) complicated
  - Provides sub-grid interface resolution

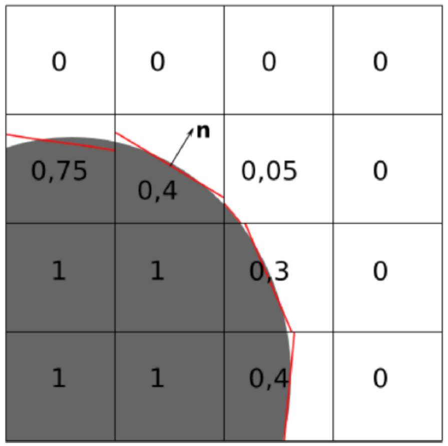


[Scardovelli & Zaleski 1999]

Interface Tracking Methods

Volume of Fluid (VOF)

wikipedia



Phase interface represented by volume fraction ψ in each cell, phase interface movement resolved by solving PDE

$$\frac{\partial \psi}{\partial t} + \nabla \psi = 0$$

However, standard convection schemes introduce too much dispersion, problem solved by artificial compression to preserve jump of ψ.

$$\psi(\mathbf{x},t) = 0 \quad \text{(gas phase)}$$

$$\psi(\mathbf{x},t) = 1 \quad \text{(liquid phase)}$$

$$0 < \psi(\mathbf{x},t) < 1 \quad \text{(2 phases present, interface)}$$

1) Reconstruction of interface geometry
   - Calculate normal ***n*** to interface
   - Assume plane interface in each cell
   - Find plane normal to ***n*** which has liquid cell volume ψ

Density $\rho = \psi \rho_l + (1-\psi)\rho_g$

Droplet Transport Modeling

Interface Tracking Methods

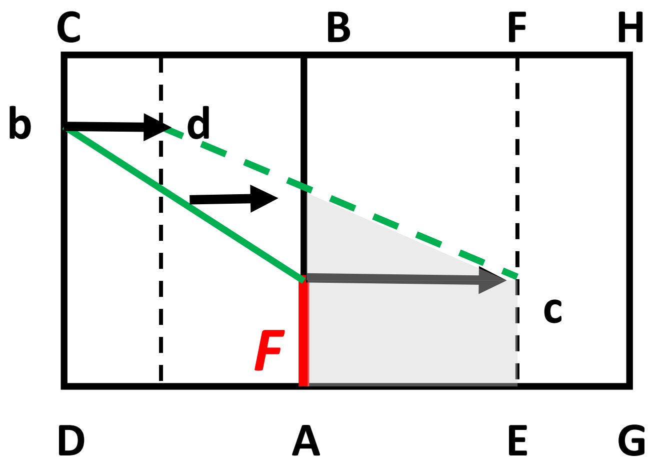
Volume of Fluid (VOF)

2) Geometric flux calculation

- Eulerian:

$$\text{flux} = \int_F \int_{t^n}^{t^{n+\Delta t}} \psi \mathbf{u} \cdot \mathbf{n} \, dt dF$$

F, wetted cell surface area;
n, cell surface normal



- Lagrangian:
  - Perform directional operator splitting
  - Advect planar interface by linearly interpolated velocities in each cell
  - Calculate change in liquid volume in cell and neighbors
  - CFL number < 0.5 (Courant-Friedrichs-Lewy number: if CFL > 1 information travels across grid cell within 1 time step → inaccurate solution)

$$C = \frac{U\Delta t}{\Delta l}$$

U, velocity;
Δt, time step;
Δl, grid size

VOF problems in practical applications

- Not exactly volume preserving
- $\Psi > 1$ or $\Psi < 0$ possible
- Low order geometric interface reconstruction
- Interface curvature not easily available (height function approach)
- Combination of geometric flux calculation and normal interface movement difficult

Droplet Transport Modeling

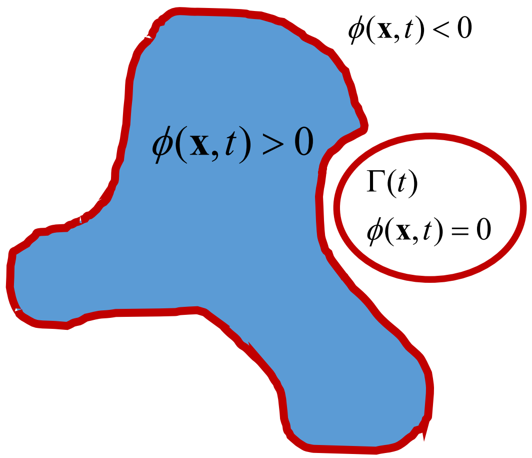
Level Set Method:

Basic idea is to follow the interface as it moves in time and space. The interface defined implicitly as iso-surface of a smooth function, *Φ*, called zero level set function. Traditionally, it is defined as a signed distance

$$\left|\nabla \phi(\mathbf{x},t)\right| = 1$$

$$\left|\phi(\mathbf{x},t)\right|$$ Gives shortest distance from ***x*** to curve

$$\phi(\mathbf{x},t) < 0 \quad (\text{gas})$$
$$\phi(\mathbf{x},t) = 0 \quad (\text{interface})$$
$$\phi(\mathbf{x},t) > 0 \quad (\text{liquid})$$



To obtain the surface *Φ(x)* discretization on a Eulerian grid and appropriate interpolation.

It is important to know the normal $$\mathbf{n} = -\frac{\nabla \phi}{\left|\nabla \phi\right|}$$

and the curvature $$\boldsymbol{\kappa} = div\left(\frac{\nabla \phi}{\left|\nabla \phi\right|}\right)$$

Parameters can be written as function *Φ(x)*,

i.e. $$\rho(\mathbf{x}) = \rho_1 + (\varphi_2 - \rho_1)\theta(\phi(\mathbf{x}))$$

Interface Tracking Methods

Smoothing by Heavyside function over a few grid cells

$$\phi(\mathbf{x},t)=\begin{cases}0 & : d<-\varepsilon \\ 1/2+d/2\varepsilon+2/2\pi\sin(\pi d/\varepsilon) & : -\varepsilon\le d\le\varepsilon \\ 1 & : d>\varepsilon\end{cases}$$

Transport equation for the level set

$$\frac{d\phi}{dt}+\mathbf{v}\left|\nabla\phi\right|=0 \qquad \mathbf{v}=F\mathbf{n}\Rightarrow\frac{d\phi}{dt}+F\left|\nabla\phi\right|=0$$

Problems with all single equation methods are tracking and reconstruction of phase boundaries, numerical diffusion and to avoid violation of continuity equation.